A COMPUTER MODEL FOR RAPID SOLUTIONS AND VISUAL CRT DISPLAY OF RADIATION PATTERNS FOR ARBITRARILY ORIENTABLE YAGI-UDA ARRAYS OPERATING OVER LOSSY GROUND OR IN SHIP-OCEAN ENVIRONMENTS

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THESIS

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A Computer Model for Rapid Solutions and Visual CRT Display of Radiation Patterns for Arbitrarily Orientable Yagi-Uda Arrays Operating Over Lossy Ground or in Ship-Ocean Environments

by

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ABSTRACT

An arbitrarily orientable Yagi-Uda array antenna was modeled, and a computer simulation run to obtain the input impedance, gain pattern and front-to-back ratio of various arrays. The model made provisions for the antenna to be operated over either a lossy ground plane or aboard a ship in seas of specified state. Quick solution turn-around, with CRT display, enabled relatively rapid optimization of numerous arrays.

Theory, resultant optimal designs and performances, photographs, and program listing are included.

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LIST OF DRAWINGS AND PICTURES*

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*Pictures are consecutive beginning with figure 7.

TABLE OF SYMBOLS

Text	Computer	Description
α	ALPH	Array axis (boresight) elevation angle; Input; Variable during ship dynamics: see equation (39).
α_t	ALTEM	A constant; Input.
β	K	Wave number.
C	CEE	Reflection factor; see equation (7).
d_i	D	Adjacent element separation; Input.
Δ	DELTA	Observation elevation angle $(90^\circ-\theta)$; Input.
Δ'	DLPRI	Element elevation angle $(90^\circ-\theta')$; Input.
$E_{\phi T}$	EPHI	Electric field; see equation (35).
$E_{\theta T}$	ETHET	Electric field; see equation (37).
ε	EPSLN	Dielectric constant of earth; Input.
f_{MH}	F	Frequency MHz; Input.
G	G	Gain; see equation (44). G_p is peak.
γ	VAR	Sinusoidal angle, 0°-360°. See equation (40c).
h	H	Height of antenna above earth plane; Input; a variable during ship dynamics; see equation (41).
h_t	HTEMP	A constant value of height; Input.
h_i	HDBL	Separation between actual and image elements; see equation (33).
I_i	CUR	Actual element current. See equations (1), (2).
ℓ_i	LH	Element half-length.

L	LH	Element length.
L_p	LP	Primary element length. See equations (12).
L_s	LS	Secondary element length. See equations (12).
N	NE	Number of array elements.
λ	LMDA	Wavelength.
Ω	KOS1	Angle between array boresight and observation line; See equation (28).
ϕ	PHI, M	Observation angle; See figure 3; Input; M varies from 0 to 90 degrees. ϕ_p occurs at peak gain.
ϕ'	PHIPR	Element angle; See figure 5; Input.
ψ	PSI	Angle between element and observation line; See equation (18).
R_h	RH	Horizontal reflection coefficient; See equation (8a).
R_v	RV	Vertical reflection coefficient; See equation (8b).
R_h'	RHPRI	Horizontal reflection coefficient used with impedance; See equation (8a), taking $\theta=0°$.
R_v'	RVPRI	Vertical reflection coefficient used with impedance; See equation (8b), taking $\theta=0°$.
σ	SIGMA	Conductivity of earth; Input.
S_z'	SZ	See equation (11a) and figure 2.
S_y'	SY	See equation (11b) and figure 2.
θ	THET, KAY	Observation angle; See figure 3; Input; KAY varies from 0 to 360 degrees. θ_p occurs at peak gain.
θ_t	THTEM	A constant value of θ; Input.

θ'	THEPR	Element angle; See figures 2,5; Input.
y_i	WYE	Separation between antenna reflector and the i^{th} element; See figure 1.
Y_o	Y0	Y coordinate of secondary antenna origin; See equation (10b).
Z_o	Z0	Z coordinate of secondary antenna origin; See equation (10a).
Z	Z	Impedance value.
$[\hat{Z}]$	ZZPAK	Impedance matrix; see equations (3) and (6).

I. INTRODUCTION

A. BACKGROUND OF THE STUDY

The most concise equation that describes the radiated electric field of an antenna assumes that the antenna operates either in free space or over a perfectly conducting ground plane. Equations in this category are simple ones that have been typically used in text books, and voluminous experimental data have been obtained through their use. Emphasis has been upon simplification of equations so that problems could be solved manually with a minimum of rigor. Equations such as these are admittedly inaccurate because they omit the component of radiation produced by ground reflection or if not omitted the ground surface is considered to be a perfect reflector.[1]

In reality the simplest antenna operates over a lossy ground and established at least a direct and a ground reflected wave. When simple free space equations are modified for two path propagation over a lossy ground plane and

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[1] The ground wave is composed of a space component and a surface component. A particularly important point may be made that for the horizontally polarized wave in the VHF/UHF range the strength of the space wave greatly exceeds that of the surface wave so as to render the surface wave negligible. This then allows an accurate field strength to be calculated using equations which assume only a two-path propagation model--the direct wave and the lossy ground-reflected wave. Henry R. Reed and Carl M. Russell, UHF Propagation, (New York: John Wiley & Sons, Inc., 1953), p. 174.

solutions are obtained by computer with a Cathode Ray Tube (CRT) display of the gain patterns, interesting results accrue.

B. STATEMENT OF THE PROBLEM

The initial problem is to obtain and verify the equations required to describe the electric field of a multi-element linear array, particularly of the Yagi-Uda type. A recent work which provided graphic computer solutions to single element antennas is the basis of this study, and as such this study is a follow-on.[2] Therefore effort centers around adapting the existing program to meet the requirements of both antenna types.

C. OBJECTIVES

The objective of this study is to provide a near real-time computer graphic solution of the gain pattern of an arbitrarily oriented Yagi-Uda array which is centered above two types of planes (with specified ε & σ):

1. the lossy ground plane, and
2. the lossy ocean plane which rolls and pitches the antenna as specified by the sea state.

Examples of gain patterns and other output parameters are illustrated.

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[2] R. W. Adler and C. B. Robbins, "The Solution and Graphic Display of Gain and Patterns for Wire and Linear Antennas in the Presence of Lossy Ground", Electrical Engineering Department, Naval Postgraduate School, to be published .

D. SCOPE AND LIMITATIONS

The following assumptions have been made for the study of the Yagi-Uda array:

1. Propagation is confined to two paths.
2. The ship-ocean model does not make any provision to augment the number of wave paths that turbulent seas might produce.
3. The current on the elements is distributed sinusoidally, e.g. the elements are thin ($d \leq \lambda/100$).
4. The elements may be spaced arbitrarily with arbitrary lengths. Thickness can be changed. Assignment of the element lengths is constrained by the fact that if elements are one wavelength then the solutions become indeterminant.

For purposes of testing the resulting equations, various array designs were attempted. The dimensions of the array were varied using the method of iterative search to uncover optimal horizontal and vertical designs of arrays with and without a reflector.

Optimality is determined by three criteria:

1. Input Impedance (Z_{in})
   Where it was possible to do so, the array was designed to have a reasonably high resistance, e.g. $\geq 20\Omega$ and a reasonably low reactance, e.g. $\leq 10\Omega$.
2. Front to Back Ratio (FBR):
   Within a satisfactory impedance range, the FBR is

maximized.[3] This expression differs in some cases from the expression for FBR that is typically used.

3. Power Gain (G):

   Finally, within the maximum FBR the gain is maximized.[4]

Photographs are shown of the linear and logarithmic results that are obtained from a variety of Yagi-Uda designs placed at different heights above ground and ocean environments.

Preliminary tests were made which showed that gain varied with change in element thickness, however for the results obtained throughout this study the element thickness to element length ratio remained fixed at 1/200.

Detailed operating procedures are found in Appendix A.

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[3] For purposes of the study the following expression for FBR isused

$$FBR = \frac{G(\theta,\phi)_{max} \;,\; 0^{\circ} \leq \phi \leq 180^{\circ}}{G(\theta,\phi)_{max} \;,\; 180^{\circ} \leq \phi \leq 360^{\circ}} .$$

This differs completely in some cases from the expression that is typically used:

$$FBR = \frac{G(\theta,90)}{G(\theta,270)} .$$

[4] The power gain used is measured with respect to an isotropic source.

II. THEORY OF THE MODEL

A. ARRAY IMPEDANCE AND ELEMENT CURRENT

The radiation problem requires the solution of the individual element currents, which will become the basis for calculating the gain of the array. The assumptions are that the driven element is excited by one volt, and that the antenna is thin and therefore the element currents are sinusoidal. The equations for the current column matrix and for $I_{max\ i}$ are

$$[I] = [\hat{Z}]^{-1} [\hat{V}] \tag{1}$$

$$I_{max\ i} = \frac{I_i}{\sin \beta \ell_i} \tag{2}$$

$[\hat{Z}]$ is the combination of the free space impedance and image impedance which is found by

$$[\hat{Z}] = [Z] + c\,[Z'] \tag{3}$$

$[Z]$ and $[Z']$ are NxN dimensional--$[Z']$ being the mutual impedance matrix relating the actual and image elements as indicated in figure 1. It should be noted that the matrices are complex. Complex inversion presents no particular problem except that the $[\hat{Z}]$ matrix must be arrayed according to the basic arithmetic operations that follow:

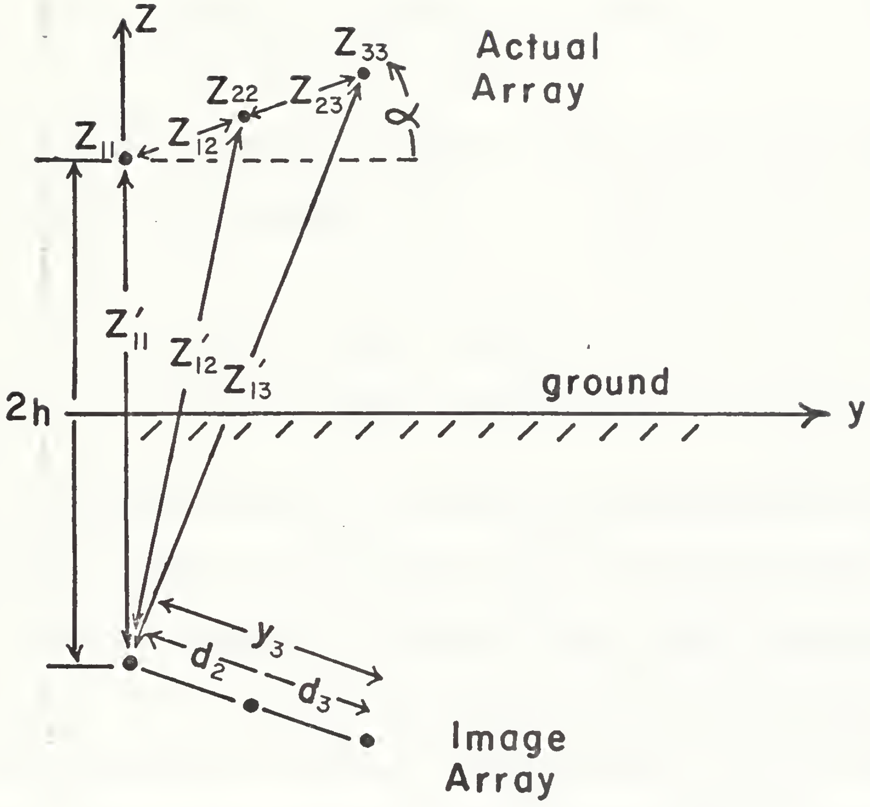

Figure 1. Side view of a 3 element Yagi-Uda array tilted α degrees above the ground.

$$([R] + j[X]) \cdot ([I_R] + j[I_I]) = [\hat{V}] \qquad (4)$$

which can be written as two equations

$$\begin{aligned} [R][I_R] - [X][I_I] &= [\hat{V}]_{Re} \\ [X][I_R] + [R][I_I] &= [\hat{V}]_{Im} \end{aligned} \qquad (5)$$

which can be solved according to equation (1), where equation (3) now appears as:

$$[\hat{Z}] = \begin{bmatrix} [R] & [-X] \\ [X] & [R] \end{bmatrix} \qquad (6)$$

$[\hat{Z}]$ is a 2Nx2N matrix. An array of maximum size (5 elements) requires inversion time on the SDS-9300 computer of 5 seconds--roughly one-eighth the total solution time needed for the complete aximuth and elevation patterns.

The reflection factor C in equation (3) is a function of the angle which the antenna element makes with the ground. C is written as[5]

$$C = e^{-j\Delta'}(R_h'\cos \Delta' + jR_v'\sin \Delta'). \qquad (7)$$

The reflection coefficients R_h' and R_v' are the values obtained when $\theta=0^\circ$ is substituted into equation (8). Theta

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[5] M. T. Ma and L. C. Walters, Power Gains for Antennas over Lossy Plane Ground, ESSA Technical Report. ERL 104-ITS 74 (U. S. Government Printing Office, Washington, D.C., 1969).

is equal to zero because the coupling which takes place between the free-space and the image element (thereby producing a mutual impedance Z') occurs with the image directly beneath the actual element. Equations (3) and (7) show that the horizontally polarized field ($\Delta'=0^\circ$) and the vertically polarized field ($\Delta'=90^\circ$) give weight to the value of [Z'] by the values $C=R_h'$ and $C=R_v'$ respectively. Since the solution to equation (3) is actually independent of θ and ϕ, one solution for this equation satisfies the gain expression at any position of observation.

The values of R_h and R_v, which are also used in the gain equation, are a function of the observation angle θ as follows:[6]

$$R_h = \frac{\cos\theta - \frac{\beta_2}{\beta} A}{\cos\theta + \frac{\beta_2}{\beta} A} \qquad (8a)$$

$$R_v = \frac{\cos\theta - \frac{\beta}{\beta_2} A}{\cos\theta + \frac{\beta}{\beta_2} A} \qquad (8b)$$

where

$$A = [1 - (\frac{\beta}{\beta_2} \sin\theta)^2]^{\frac{1}{2}} \qquad (9a)$$

$$\beta_2 = \beta[\varepsilon_r - j \frac{\sigma}{\omega\varepsilon_0}]^{\frac{1}{2}} \qquad (9b)$$

or

$$\beta_2 = \beta[\varepsilon_r - j \frac{1.8\ \sigma\ 10^4}{f_{MH}}]^{\frac{1}{2}} \qquad (9c)$$

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[6] Ibid.

The matrix [Z'] is identical to equation (6) except that it is written in terms of [R'] and [X']. The geometric orientation of the antenna elements are central to the solution for the mutual impedance matrix [Z']. Figure 2 shows an arbitrarily oriented single element and its corresponding image in the ground plane. Although the true coordinates of the antenna are in the xyz coordinate system, the equation for solving for the impedance fixes the primary element along a vertical axis--here it is shown to be z''. The secondary element becomes the image element and is separated from the primary element by the distances Z_o and Y_o. The linkage between the two coordinate systems xyz and z'y'z' is θ'. The equations for Z_o and Y_o are observed to be

$$Z_o = -2h \cos \theta' = -2h \sin \Delta' \tag{10a}$$

$$Y_o = 2h \sin \theta' = 2h \cos \Delta' \tag{10b}$$

$$S_{z'} = S \cos 2\theta' = S \cos 2\Delta' \tag{11a}$$

$$S_{y'} = -S \sin 2\theta' = -S \sin 2\Delta' \tag{11b}$$

The solution of self impedance is performed by assuming that $Z_o=0$ and $Y_o= 2\ r$, where the radius r is assumed equal to $\ell_i/200$. (This makes $\ell_i/d = 100$, which for $\ell_i = \lambda/4$ satisfies the thin antenna specifications of $d \leq \lambda/100$). The mutual impedances of the free-space array, where the elements are all parallel, are solved using $Z_o=0$ and $Y_o=y_i$, the separation between the two elements in question. Under dynamic ship motion, which affects θ' and h, the antenna height above the earth plane, the mutual impedance

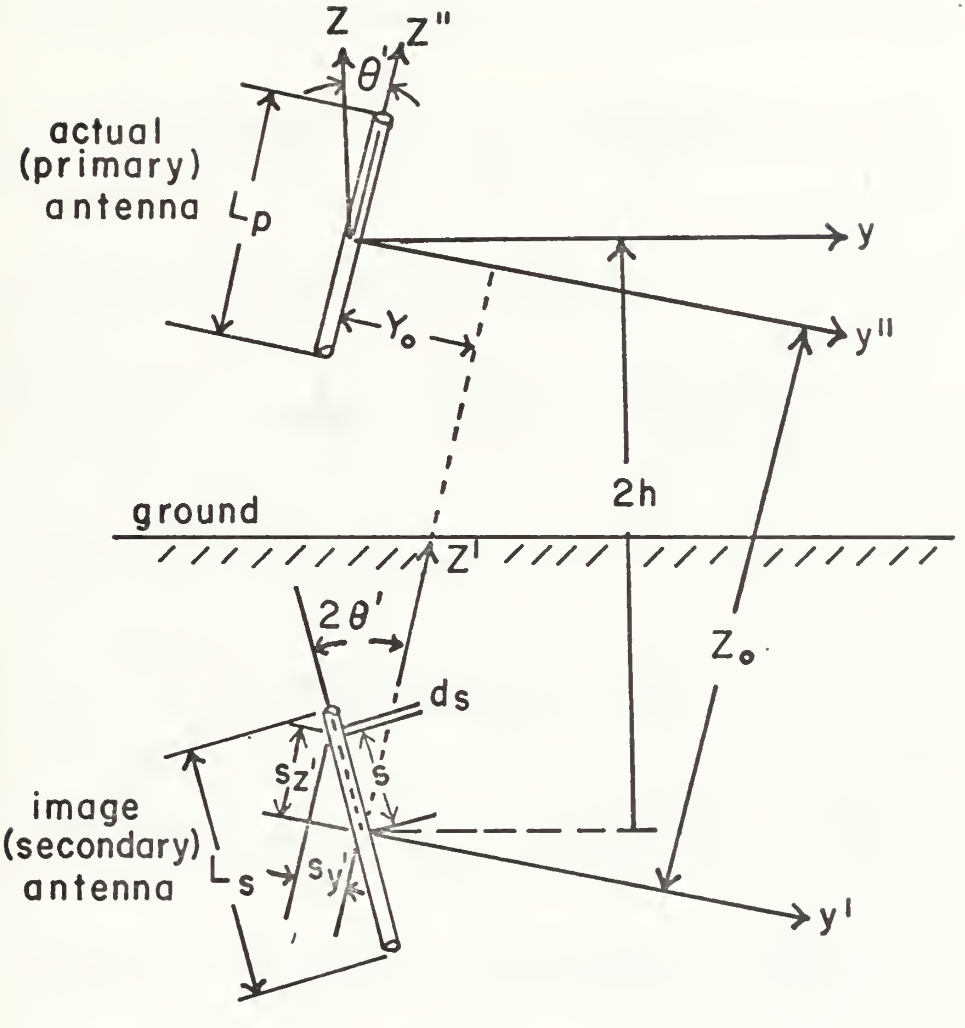

Figure 2. The geometry of an arbitrarily oriented element in free space and its image (used for solution of mutual impedances Z')

matrix [Z'] will be affected according to equations (10) and (11), but the self and mutual impedances of the [Z] matrix will not be affected since it represents the free-space array impedance.

Impedances are solved according to the following:[7]

$$R = -30 \int_{-L_s/2}^{L_s/2} \left\{ \left[\frac{\sin(2\pi r_1)}{r_1} \left(Z_o + S_z' + \frac{L_p}{2\lambda}\right) + \frac{\sin(2\pi r_2)}{r_2} \left(Z_o + S_z' - \frac{L_p}{2\lambda}\right) \right. \right.$$

$$\left. - \frac{2 \sin(2\pi r)}{r} \cos\left(\frac{\pi L_p}{2\lambda}\right) (Z_o + S_z') \right] \frac{S_y'}{Y + S_y'}$$

$$+ \left[\frac{2 \sin(2\pi r)}{r} \cos\left(\frac{\pi L_p}{\lambda}\right) - \frac{\sin(2\pi r_1)}{r_1} - \frac{\sin(2\pi r_2)}{r_2} \right] \cdot S_z'$$

$$\left. \cdot \frac{\sin\left(2\pi \frac{L_s}{2\lambda} - |s|\right)}{s} \right\} ds \tag{12a}$$

$$X = -30 \int_{-L_s/2}^{L_s/2} \left\{ \left[\frac{\cos(2\pi r_1)}{r_1} \left(Z_o + S_z' + \frac{L_p}{2\lambda}\right) \right. \right.$$

$$+ \frac{\cos(2\pi r_2)}{r_2} \left(Z_o + S_z' - \frac{L_p}{2\lambda}\right) - \frac{2 \cos(2\pi r)}{r} \cos\left(\frac{\pi L_p}{\lambda}\right)$$

$$\left. (Z_o + S_z') \right] \frac{S_y'}{(Y_o + S_y')^2} + \left[\frac{2 \cos(2\pi r)}{r} \cos\left(\frac{\pi L_p}{\lambda}\right) \right.$$

$$\left. \left. - \frac{\cos(2\pi r_1)}{r_1} - \frac{\cos(2\pi r_2)}{r_2} \right] S_z' \; \frac{\sin\left(2\pi \frac{L_s}{2\lambda} - |s|\right)}{s} \right\} ds \tag{12b}$$

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[7] H. C. Baker, and A. H. Lagrone; "Digital Computation of the Mutual Impedance Between Thin Dipoles;" IRE Transactions on Antennas and Propagation; March, 1962; AP-10, No. 2; pps 172-178.

$$\rho^2 = (Y_o+S_y')^2 \tag{13a}$$

$$r = [\rho^2 + (Z_o+S_z')^2]^{\frac{1}{2}} \tag{13b}$$

$$r_1 = [\rho^2 + (Z_o+S_z'+\frac{Lp}{2})^2]^{\frac{1}{2}} \tag{13c}$$

$$r_2 = [\rho^2 + (Z_o+S_z'-\frac{Lp}{2})^2]^{\frac{1}{2}} \tag{13d}$$

Where the distances r, r_1, and r_2, are the respective distances in figure 2 from the bottom, center and top of the primary antenna to the differential, ds, on the secondary antenna.

B. THE FAR FIELD

To solve for the electric field, a point of observation is chosen, and each element in the array is viewed. Each element contributes to the total field according to its vector phase and amplitude, its separation from some reference element, the tilt angle of the array, and the bearing to the point of observation. Figure 3 shows the geometry of a 2 element horizontal Yagi-Uda tilted α degrees.

The equation of the field produced by one element located at the origin is first developed. Then, the presence of other elements in the array is taken into account.

The equation at the point P for an element at the origin is

$$I_i = I_{o_i} \sin[\beta(\ell_i \pm \chi)]\, e^{j\omega(t-\frac{S}{c})} \tag{14}$$

Current flowing in the incremental length dx produces a field, and there will be a phase associated with it

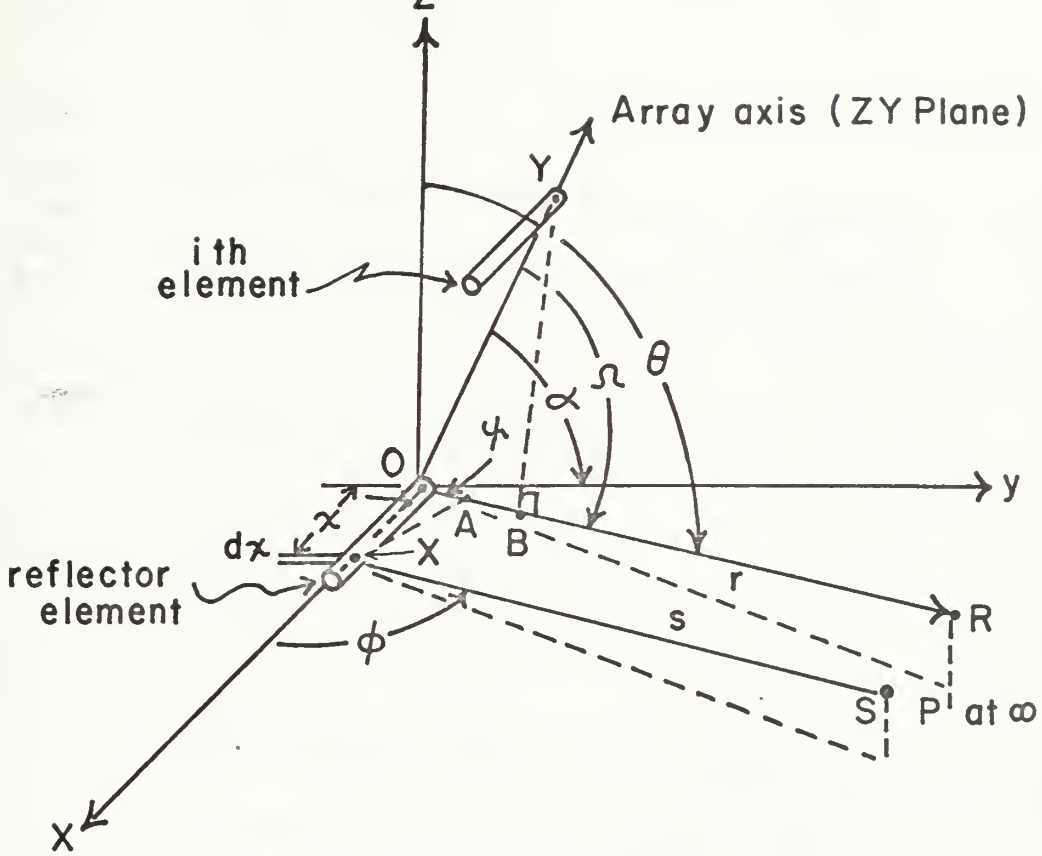

Figure 3. Yagi-Uda array with horizontal elements and a tilted axis, as viewed from a distant point P. $(\phi'=0, \theta'=90)$

with respect to point P as the differential length moves from $-\frac{L_s}{2}$ to $+\frac{L_s}{2}$. The delay is $\chi \cos\psi$, which is found by the dot product of $\widehat{OX}$ and $\widehat{OR}$, as follows:

$$OA = \widehat{OX} \cdot \widehat{OR}$$

where.

$$\widehat{OX} = \chi[\hat{i}\sin\theta'\cos\phi' + \hat{j}\sin\theta'\sin\phi' + \hat{k}\cos\theta'] \quad (15a)$$

$$\widehat{OR} = [\hat{i}\sin\theta\cos\phi + \hat{j}\sin\theta\sin\phi + \hat{k}\cos\theta] \quad (15b)$$

so that

$$OA = \chi[\sin\theta\sin\theta'\cos(\phi-\phi') + \cos\theta\cos\theta'], \quad (16)$$

or

$$OA = \chi\cos\psi \quad (17)$$

where

$$\cos\psi = \sin\theta\sin\theta'\cos(\phi-\phi') + \cos\theta\cos\theta'. \quad (18)$$

It is now clear that $s = r - \chi\cos\psi$ and

$$I_i = I_{o_i}\sin[\beta(\ell_i \pm \chi)]\, e^{j\omega[t-(\frac{r-\chi\cos\psi}{c})]} \quad (19)$$

where

$$\ell \pm \chi = \begin{cases} \ell + \chi, & \chi < 0 \\ \ell - \chi, & \chi > 0 \end{cases} \quad . \quad (20)$$

Although there is a phase difference between r and s, at points taken far from the origin their magnitudes are approximately equal.

Observe that by translating the xyz origin as well as the off-axis element to the point B located along the

observation vector $\hat{r}$, the magnitude of the new observation vector relative to the new origin is $\widehat{BR}$, which differs in length from the old observation vector $\widehat{OR}$. The amount of change equals that which affects the observation vector $\hat{s}$. Furthermore the angle by definition remains unchanged. Therefore, the expression $r - s = \chi \cos \psi$ holds for every radiating element in the array, not just for the element at the origin. From this it can be concluded that the integral equation for the field is identical for each element in the array.

Following the method used by Kraus:[8]

$$E\phi_i = \frac{j\,60\,\pi\,\sin\phi}{s\lambda}\int_{-\ell_i}^{\ell_i} I_i\,dx \quad . \tag{21}$$

Substituting for I and letting $\beta = \frac{2\pi}{\lambda} = \frac{\omega}{c}$, we see that:

$$E\phi_i = \frac{j\,30\,\beta}{r}\sin\phi\, I_{o_i}\, e^{j\omega(t-\frac{r}{c})}\int_{-\ell_i}^{\ell_i}\sin(\beta\ell_i \pm \beta\chi)\,e^{j\beta\chi\cos\psi}\,dx. \tag{22}$$

When this expression is integrated the result is:

$$E\phi_i = \frac{j\,60\sin\phi\, I_{o_i}\, e^{j\omega(t-\frac{r}{c})}}{r\sin^2\psi}\left[\cos(\beta\ell_i\cos\psi)-\cos(\beta\ell_i)\right] \quad . \tag{23}$$

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[8] J.D. Kraus; Antennas; (New York: McGraw-Hill Co. 1950), pp. 135-141.

For an array of elements the field is the superposition of the fields of each element taken separately. E_ϕ is then the sum of $E\phi_i$ given by

$$E_\phi = \frac{e^{-j\beta r}\ 60\ \sin\phi}{r\ \sin^2\psi} \sum_{i=1}^{N} I_{o_i}\ [\cos(\beta\ell_i\cos\psi) - \cos(\beta\ell_i)]. \quad (24)$$

It is recognized that the time term, $e^{-j\omega t}$ has been dropped and that β has been substituted for ω/c.

The array factor, or the phase delay $\hat{OB}$ introduced by the separation between elements, is next taken into account. From figure 3, using the reflector as reference, the separation between elements as viewed from point P is $OB = \hat{OY} \cdot \hat{OR}$, where

$$\hat{OY} = \hat{j} y_i \cos\alpha + \hat{k} y_i \sin\alpha \quad (25)$$

and $$\hat{OR} = \hat{i}\ \sin\theta\ \cos\phi + \hat{j}\ \sin\theta \sin\phi + \hat{k}\ \cos\theta \quad (26)$$

gives $$OB = y_i(\cos\alpha\ \sin\theta\ \sin\phi + \sin\alpha \cos\theta). \quad (27)$$

If $$\cos\Omega = \cos\alpha\ \sin\theta\ \sin\phi + \sin\alpha\ \cos\theta \quad (28)$$

then $$OB = y_i\ \cos\Omega \quad (29)$$

represents the phase lead of the i^{th} element.

Having found the slant separation between elements to be expressed by equation (29), the current for any element in the array may be written as

$$I_{o_i} = I_{Max\ i}\ e^{j\beta y_i \cos\Omega}. \quad (30)$$

The phasor sum of currents in the array becomes

$$I_o = \sum_{i=1}^{N} I_{max\ i}\ e^{j\beta y_i \cos \Omega} . \tag{31}$$

Upon substitution of equation (31) into equation (24) the result is

$$E_{\phi_i} = \frac{j\ 60\ e^{-j\beta r} \sin \phi}{r \sin^2 \psi} \sum_{i=1}^{N} I_{max\ i}\ e^{j\beta y_i \cos \Omega} [\cos(\beta \ell_i \cdot \cos \psi) - \cos(\beta \ell_i)] \tag{32}$$

where $I_{max\ i}$ is found from equation (2).

Equation (32) represents the free space radiation of the horizontally polarized Yagi-Uda. The equation does not consider the ground reflections. Figure 4 shows the direct and ground reflected waves, and the geometry shows the difference between their path lengths. The element separation, written as:

$$2h_i = 2(h + y_i \sin \alpha) \tag{33}$$

is a function of the path difference vector $\hat{AB}$, since $AB = 2h_i \cos \theta$.

Thus

$$E_{\phi_T} = E_\phi + R_h E_\phi\ e^{-j2\beta h_i \cos \theta} . \tag{34}$$

And finally, equations (32) and (34) are combined to produce the final space wave in the phi direction:

$$E_{\theta_T} = \frac{j\ 60\ e^{-j\beta r} \sin \phi}{r \sin^2 \psi} \sum_{i=1}^{N} I_{max_i} [1 + R_h\ e^{-j2\beta h_i \cos \theta}] \cdot e^{j\beta y_i \cos \Omega} [\cos(\beta \ell_i \cos \psi) - \cos(\beta \ell_i)] . \tag{35}$$

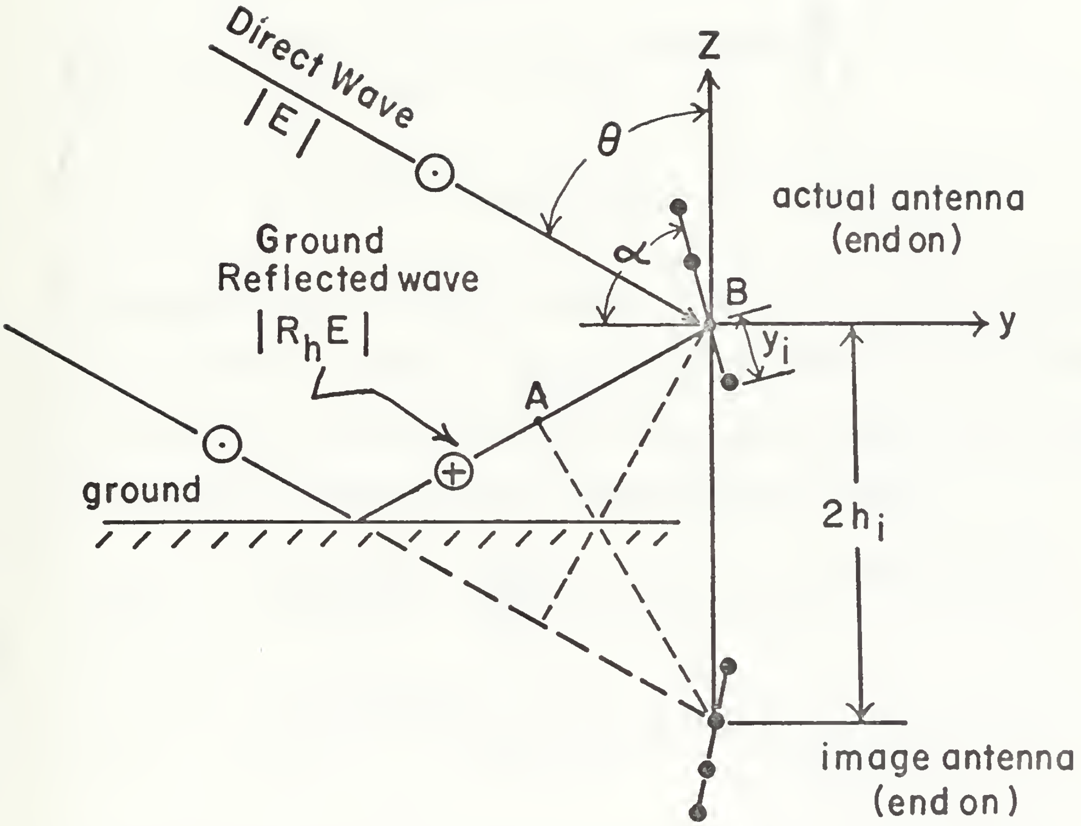

Figure 4. The tilted Yagi-Uda receiving horizontally polarized waves via direct and reflected paths. (The array is tilted to present less confusion in distinguishing angles)

Observe that when $\cos\Omega = \sin\theta \sin\phi$ and $h_i=h$ equation (35) agrees with Ma and Walters equation for the horizontal Yagi-Uda.[9] The equation for the θ component of the horizontal Yagi-Uda, E_θ, is not derived with the same rigor since it represents the cross polarized radiation, which exists only off-axis and is of minor interest. The equation is:

$$E_{\theta_T} = \frac{j\,60\,e^{-j\beta r}\cos\theta\cos\phi}{r\sin^2\psi} \sum_{i=1}^{N} I_{\max_i}\left[1 - R_v e^{-j2\beta h_i\cos\theta}\right] e^{j\beta y_i\cos\Omega}\left[\cos(\beta\ell_i\cos\psi) - \cos(\beta\ell_i)\right]. \qquad (36)$$

The equation for the vertical dipole is well known.[10] If R_v is used insteady of R_h then equations (31) and (34) may be applied to the standard free space equation to arrive at the general solution for the vertical array:

$$E_{\theta_T} = \frac{j\,60\,e^{-j\beta r}}{r\sin\theta} \sum_{i=1}^{N} I_{\max_i}\left[1 + R_v\,e^{-j2\beta h_i\cos\theta}\right] e^{j\beta y_i\cos\Omega}\left[\cos(\beta\ell_i\cos\psi) - \cos(\beta\ell_i)\right] \qquad (37)$$

Specifically, $\cos\psi = \sin\theta\sin\phi$ and $\cos\Omega = \cos\theta$ since $\alpha = \theta = 0$ for the vertical Yagi-Uda.

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[9] M.T. Ma and L.C. Walters, op.cit. p. 41.

[10] Kraus, loc.cit.

C. THE SHIP-OCEAN MODEL

Figure 5 shows the effects of the dynamic ship model upon the Yagi-Uda. Changes in the sinusoidal waves, considering that a roll of eight degrees per sea-state is produced and a bow pitch of three degrees per sea-state is produced (representative of a light cruiser), will affect the angles θ' and α as follows:

$$\theta' = \theta_t' - (\Delta_1 \cos \phi' + \Delta_2 \sin \phi') \tag{38}$$

$$\alpha = \alpha_t - \Delta_1 \sin \phi' + \Delta_2 \cos \phi' \tag{39}$$

where

$$\Delta_1 = \text{(wave)} \sin \text{(course)} \tag{40a}$$

$$\Delta_2 = \text{(wave)}.3 \cos \text{(course)} \tag{40b}$$

and where

$$\text{wave} = 8\text{(sea state)} \sin (\gamma) \quad . \tag{40c}$$

The effective height of the array will change according to

$$h = h_t \cos \Delta_1 \cos \Delta_2 \tag{41}$$

It should be noticed that the parameter ϕ' until now has been somewhat extraneous for the solution to antennas over land, since variation of the observation angle ϕ' and the angle ϕ accomplish the same thing. However the angle ϕ' now is significant in specifying the antenna orientation with respect to the ship heading, which then determines the aspect that the wave presents to the antenna.

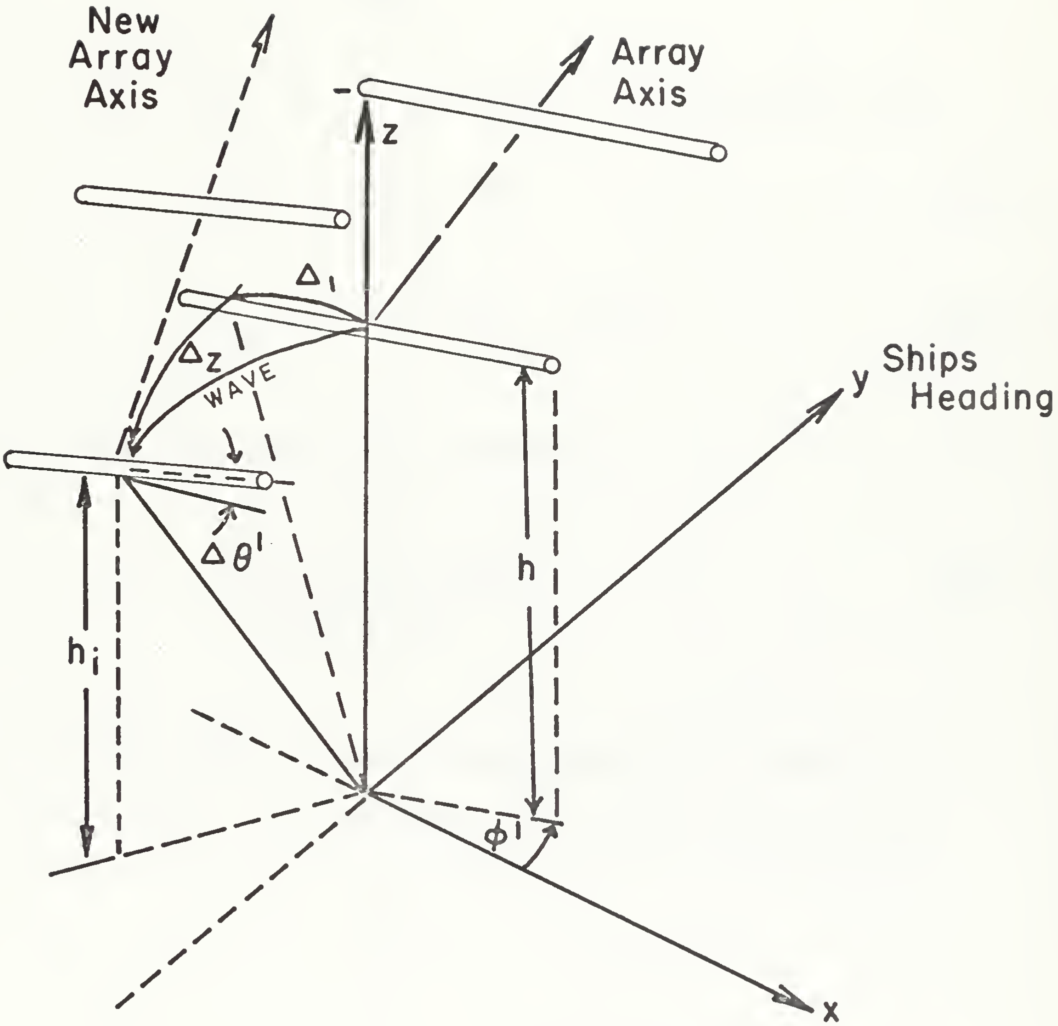

Figure 5. Orientation of the Yagi-Uda aboard a ship. Angles Δ_1 and Δ_z represent roll and bow pitch respectively.

D. GAIN

The gain of an antenna is equal to the ratio of the power intensity to the power density, and is expressed as follows:

$$G = \frac{4\pi W_r}{W_{in}} \tag{42}$$

where $W_r = r^2 \, (\hat{E} \times \hat{H}^*)$ watts/solid angle

or

$$W_r = \frac{r^2}{120\pi} \, |E|^2 \tag{43a}$$

and

$$W_{in} = |I_b|^2 \, R_{in} \quad \text{watts} \; . \tag{43b}$$

The final expression for gain appears as

$$G = \frac{r^2 |E_{\theta T}|^2 + |E_{\phi T}|^2}{30 \, |I_b|^2 \, R_{in}} \; . \tag{44}$$

The equation for gain becomes independent of r when the equations (35) and (36) are substituted into the equation above.

III. RESULTS OF ARRAY PERFORMANCE

A. RESULTS OVERVIEW

Basically, four types of arrays were examined for optimization: horizontal and vertical with and without reflectors. Experiments were conducted to determine the design of arrays that afford optimal performance as defined in the Scope and Limitations section of the study. Starting with the two element array, the parasitic element functioning both as a director and as a reflector, the element spacings and lengths were varied to obtain optimization. The arrays were lengthened by adding one element at a time, up to a total of five elements, each time solving for an optimal design. Numerous iterations were required, and the time required to optimize an array varied from ten minutes for a two element array to slightly over three hours for a five element array. (The vertical array was more difficult than the horizontal, and the array without reflector was more difficult than that with reflector.) Not all of the multi-element arrays were optimized according to the rigid criteria chosen for this study. Specifically, the vertical designs having four and five elements were arrived at by relaxation of the impedance threshold criterion thereby producing a suboptimal design.

B. THE TWO ELEMENT ARRAY

Referring to Table I for the horizontal two-element array with a director, maximum FBR is maintained when, for

TABLE I

OPTIMAL 2-ELEMENT HORIZONTAL ARRAY AT VARIOUS ELEMENT SPACINGS USING THE PARASITE AS A REFLECTOR AND AS A DIRECTOR

(Gain and Front-to-Back ratio in db)
(Spacing and length in λ)

	d_2	.012	.100	.150	.200	.251	.295
DIRECTOR	G_p	10.3	10.6	10.1	9.5	8.7	7.8
	FBR_p	13.3	11.9	6.7	3.7	1.9	0.8
	L_{d_i}	.492	.460	.449	.438	.424	.390
REFLECTOR	G_p	10.5	10.9	10.7	10.5	10.1	9.9
	FBR_p	10.9	13.6	13.6	13.3	12.5	11.9
	L_{r_i}	.501	.501	.501	.501	.506	.501

$f_{MH}=146$. h=1m. In both cases the length of the driven element is λ/2. (L_d and L_r are director and reflector lengths respectively.) (The subscript p denotes peak, e.g. the length of the parasitic element was varied until the peak FBR was first obtained, then within that value, the gain peak was found by making finer adjustments to length.)

increasing element spacing, there is a corresponding decrease in director length. Apparently, best performance occurs when spacing is .01λ (.021m) because this is where the maximum FBR appears. However, no mention has been made about the affect upon impedance of varying the element spacing.

In general it was found that the resistance and inductive reactance decreased together (from 70 to 40 ohms and from 40 to 10 ohms respectively) as elements were placed closer together. When the spacing closed to .05λ, the currents in the elements evidently became very large, and the value of resistance became less than one ohm (too low for convenient matching). Thus the FBR and G are insufficient criteria of optimality since impedance can fall (in some cases may rise) to unreasonable values. Optimality in the case of the director array in Table I actually occurs at .1λ (.205m) spacing.

When the parasitic element is used as a reflector it is not necessary to change the reflector element length as the spacing between elements is changed in search for the maximum FBR. The maximum FBR occurs at .5λ (1.03m) for all spacings except at .25λ (.515m); that is, the maximum FBR occurred when both element lengths were the same.

The gain curves shown in figure 6 differ from other published experimental and theoretical data (and there are fairly wide variations in these results too), inasmuch as the value of the peak gain is somewhat higher than that typically recorded. This is because published data typically

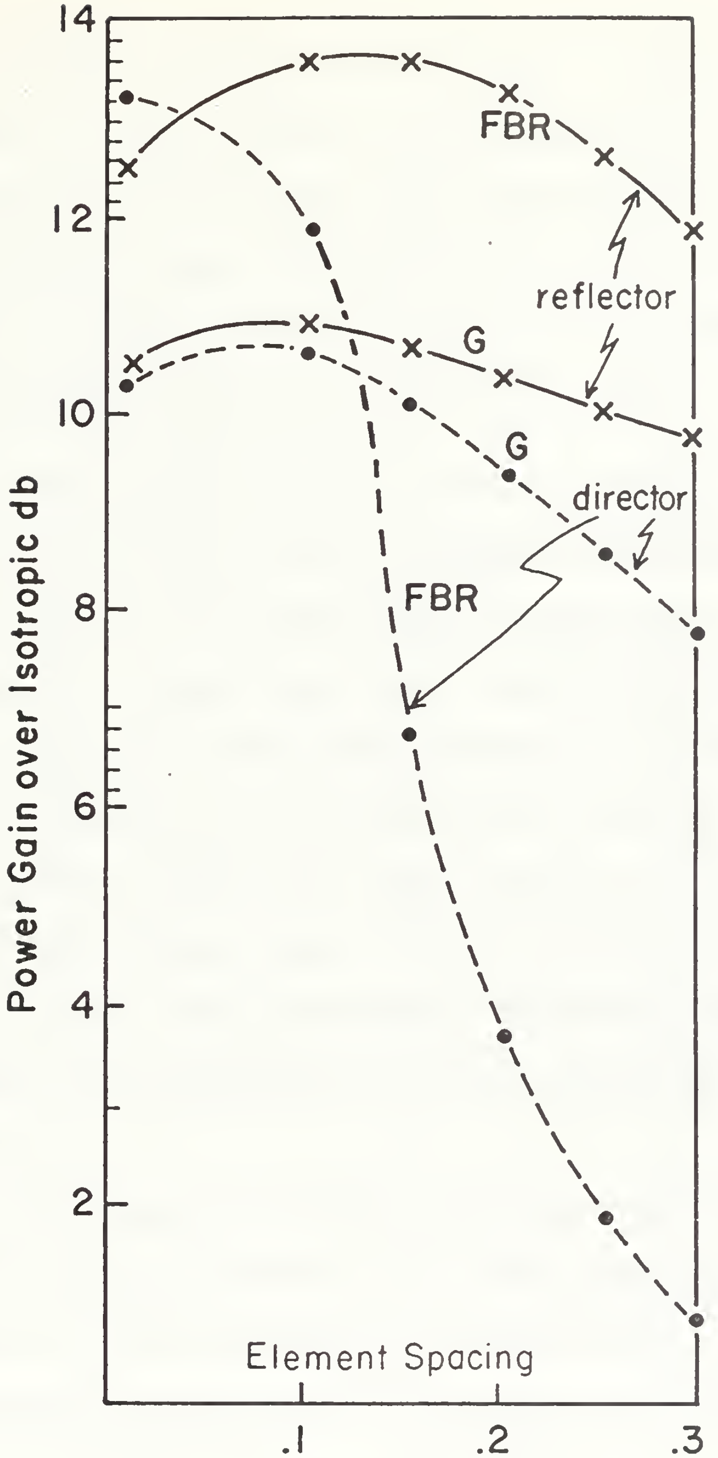

Figure 6. Comparison of gain vs element spacing for optimal 2-element horizontal arrays. These curves represent data in table 1, where the array is a reflector—driven element combination in one case and a director-driven element combination in the other case.

does not reflect a lossy ground plane (two-path propagation is ignored). Also, the gain of the reflector array exceeds that of the director array, which generally disagrees with other findings. Again, the explanation for these differences is tracable to the assumption that a lossy ground wave is being propagated along with the direct wave.[11]

C. THE MULTI-ELEMENT VERTICAL AND HORIZONTAL ARRAYS

Tables II and III list the optimal and suboptimal design parameters for the $f_{mh}=30$ arrays. Notice that the four and five element vertical arrays have relatively wide reflector spacings--these are suboptimal designs. By comparison with the optimal design that was found for the four element vertical array (where $R_{in} \geq 20\Omega$) the FBR is 16.4 db whereas the suboptimal design ($R_{in} < 20\Omega$) produced a FBR of 22.4 db -- quite a drop. The two designs differed completely.[12] Furthermore they responded to slight parameter variations differently.

The four element vertical array was fine-tuned for maximum FBR only to discover that the impedance was unacceptable; all subsequent fine adjustments, introduced for purposes of raising the resistance to a value above 20Ω,

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[11] C. R. Fry, The Yagi-Uda Aerial--A Short Design Review and Bibliography, (Valcartier, Quebec: Canadian Armament Research and Development Establishment, May 1966), p. 23.

[12] The optimal design, as compared with the suboptimal design shown in Table II, was $L_i=.49,.47,.45,.43$; $d_i=.22,.19,.15$ (L and d in λ).

TABLE II

PARAMETERS OF OPTIMAL 30 MHz HORIZONTAL AND VERTICAL ARRAYS WITH REFLECTOR.

(spacing and Lengths in λ)

		2-Element	3-Element	4-Element	5-Element
HORIZ	L_i	.51 .47	.51 .50 .46	.51 .475 .41 .42	.51 .48 .40 .42 .43
	d_i	.11	.25 .08	.16 .10 .10	.16 .06 .06 .06
VERT	L_i	.51 .47	.49 .47 .45	* .50 .49 .47 .49	* .50 .49 .47 .42 .30
	d_i	.11	.20 .20	* .36 .09 .28	* .41 .09 .3 .25

$h=30$m, $\epsilon=5$, $\sigma=10^{-3}$, $\phi=90$. (horiz $\theta=85^\circ$. & vert $\theta=86^\circ$.)

*THESE DESIGNS ARE ACTUALLY SUBOPTIMAL

TABLE III

RESULTS OF OPTIMAL 30 MHz HORIZONTAL AND VERTICAL ARRAYS WITH REFLECTOR AND OPTIMAL HORIZONTAL ARRAY ORIENTED VERTICALLY

(Specifications and parameters are same as in Table II)
(Gain and Front-to-Back Ratio in db)
(R and X in ohms)

	OPTIMAL ARRAY WITH REFLECTOR								OPTIMAL HORIZONTAL @ Vert Orientation			
	Horiz @ Horiz Orientation				Vert @ Vert Orientation							
N	2	3	4	5	2	3	4	5	2	3	4	5
G	11.8	12.6	13.3	13.5	10.5	13.2	13.5	13.6	10.5	11.3	11.9	12.2
FBR	11.6	22.0	23.9	27.3	5.3	16.1	22.4	26.6	5.3	7.2	9.5	10.3
R_{in}	27.3	31.6	36.1	28.3	23.8	20.2	18.8	16.6	27.3	31.9	35.2	28.5
X_{in}	9.3	0.1	-2.3	-1.0	7.1	-2.6	-6.1	-3.9	9.3	0	-2.1	-0.9

caused a drastic reduction in FBR--below 16 db. Thus, this particular design was abandoned; another completely different design provided an acceptable impedance with a peak fine-tuned FBR value of 16.4 db (compare parameters in footnote #12 with the comparable parameters in Table II).

Except for the two larger vertical arrays (four and five elements), all changes that were made for the purpose of increasing R_{in} or reducing X_{in} inevitably improved the FBR figure and in none of these cases did the manipulation of parameters for purposes of adhering to the impedance criterion result in obtaining a lower FBR or G value. Because such manipulation did, however, cause a drop in the FBR for the two vertical arrays they are considered anomalous.

The optimal horizontal and vertical arrays[13] have tapered elements in the region next to the driven element, but the last few end-directors of the four and five element horizontal array show a slight inverse taper. Writers, however, generally agree that elements and spacings that are gradually tapered towards the end of an array will usually give best results.

It is obvious that the optimum parameter measurements obtained for the reflector-driver combination (two-element array) do not necessarily ensure optimization of successive experiments involving the addition of another director. A

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[13] The optimal arrays include all horizontal plus the two and three element vertical arrays.

design readjustment of all parameters is required each time the array length is changed (e.g. each time an element is added or removed). A corollary to this for arrays with more than two elements is that the FBR will decrease with the addition of a director whose length and spacing are identical to those of the preceding director. This was observed in testing the uniform array[14] at $f_{MH}=10$ where the FBR dropped from 14.6 db as a four element array to 10 db as a five element array.

With regard to director spacings, perhaps it is more than just coincidental that the designs which produced optimization of the longer horizontal arrays resulted in equispaced director elements. Nothing could be obtained to substantiate whether other analysts would agree.

From Table III, comparison between the horizontal three and four element arrays as well as comparison between the four and five element arrays shows that the FBR improvement is greater than the G improvement as array lengths increase. An added advantage, besides increasing FBR and G, is that the greater the number of elements used in the optimal array the greater the tendency for automatic reactance cancellation. While the reactive components of the two and three element arrays are small they did not become so as a coincidence of optimization; manipulation of parameters was necessary to obtain reactance decreases in conjunction with

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[14] Design: L_i=.533,.500,.434,.434,.434 and d_i=.233,.100, .100, @ h=3 (L, d and h are in λ).

the need to increase resistance (and, as inferred earlier, these manipulations in the case of the two and three element arrays produced slight decreases in FBR and G[15]).

Comparison between the optimal horizontal array and its vertical counterpart indicates that the horizontal array consistently out-performs the vertical array in FBR, but matches it in G. Furthermore, when the optimally designed horizontal array is operated in the vertical position and vice versa a degradation in performance results. An improvement of almost 16 db in FBR and 1.5 db in G can be obtained when the array is optimized in the position for which it is intended to be operated.

Table IV gives the results of operating the optimal array without its reflector; it is interesting to observe the importance that the reflector has in determining the overall array performance. Practically no directivity was obtained without a reflector even with a four element (three director) array. The unusually large G that results for the three and four element vertical arrays are anomalies

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[15] Fishenden reiterates the popular belief that a convenient method of altering the input impedance without affecting FBR or G is by varying the spacing between the reflector and the driven element from λ/8 to λ/4. Such attempts in this study did however noticably affect the FBR and G (particularly in the case of the vertical four and five element arrays, which were particularly troublesome). R. M. Fishenden and E. R. Wiblin, "Design of Yagi Aerials", Proceedings IEE, 96, Pt. 3, (Jan 1949), p. 6.

TABLE IV

THIRTY ARRAYS DESIGNED AS SHOWN IN TABLE II BUT WITH REFLECTORS REMOVED

	Horiz @ Horiz Orientation				Vert @ Vert Orientation			
N	1	2	3	4	1	2	3	4
G	7.8	11.5	5.0	7.9	7.8	4.8	15.5	15.6
FBR	0	0	1.7	1.8	0	0	2.8	2.9
R_{in}	60.6	29.4	4.1	33.0	60.6	34.2	18.1	16.1
X_{in}	-12.3	-0.1	-15.3	-9.0	-12.3	-9.7	-6.4	-4.2

and cannot be explained (undoubtedly it is connected with the fact that these two arrays are actually suboptimally designed to begin with.)

Such poor performance is not without remedy, however, as the results show in Tables V and VI. Arrays without reflectors may also be optimized with considerable improvement in performance. Unlike the arrays with reflectors, the optimal design of arrays without reflectors gives identical optimal results regardless of whether the array is operated in the horizontal or vertical position. Inasmuch as the reactance is much greater than 10Ω, these arrays do not represent optimal designs according to the purposes of this study, yet none better could be found.

D. SELECTED INTERESTING PATTERNS

Numerous photographs were taken of the following antenna patterns (in a few cases comparisons between the linear and the log gain plots are shown):

1. a three element array operating over dry land, fresh water and sea water,
2. a three element array at different heights over land,
3. several three element arrays operating on a ship underway in heavy seas,
4. a three element array with its axis tilted at various angles from the horizon and
5. some resulting patterns when an array is operated off-frequency.

TABLE V

PARAMETERS OF OPTIMAL 30 MHz ARRAYS WITHOUT REFLECTOR

(Horizontal & Vertical Optimal Arrays Identical)
(L and d in λ)

		2-Element		3-Element			4-Element			
H& OV RE IR ZT	L	.49	.48	.51	.475	.475	.52	.44	.45	.43
	d	.05		.08	.10		.13	.13	.13	

TABLE VI

RESULTS OF OPTIMAL ARRAYS WITHOUT REFLECTOR

(Specifications and parameters same as in Table V)
(G and FBR in db, R and X in ohms)

N	2	3	4
G	19.7	13.6	10.1
FBR	0	14.9	26.2
R	4.8	76.6	73.9
X	-6.2	57.4	74.5

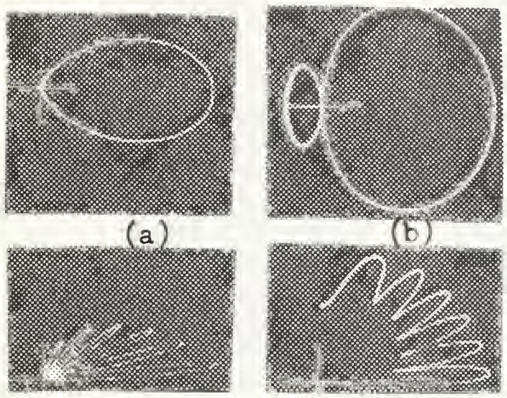


Figure 7 Optimal 30 MHz 3-element horizontal array: linear plot (a) and log plot (b).[16]

G_p = 12.6 db
FBR_p = 22.0 db
R_{in} = 31.6 Ω X_{in} = .1 Ω

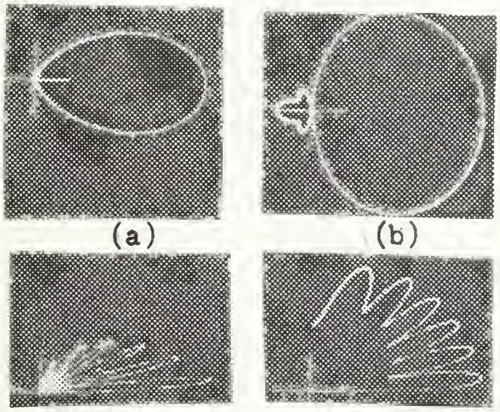


Figure 8 Optimal 30 MHz 4-element horizontal array: linear plot (a) and log plot (b).[16]

G_p = 13.3 db
FBR_p = 23.9 db
R_{in} = 36.1 Ω X_{in} = -2.3 Ω

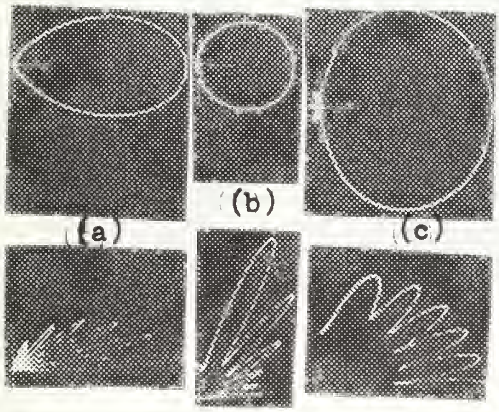
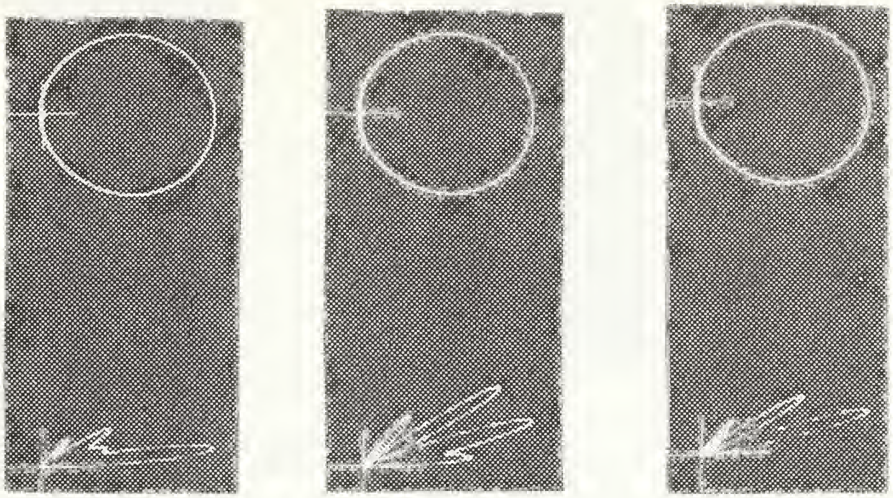


Figure 9 Optimal 30 MHz 5-element horizontal array: linear plots (a) & (b), and log plot (c).[16]

G_p = 13.5 db
FBR_p = 27.3 db
R_{in} = 28.3 Ω X_{in} = -1.0 Ω

---

[16] Parameters that are not shown in Table II but which are common to all the 30 MHz arrays are: $h_t=3$, $\varepsilon=5$. $\sigma=10^{-3}$ $\phi_p=90$. $\theta_p=85$, however figure 9 (c) differs in observation, with $\phi=20$. and $\theta=60$.

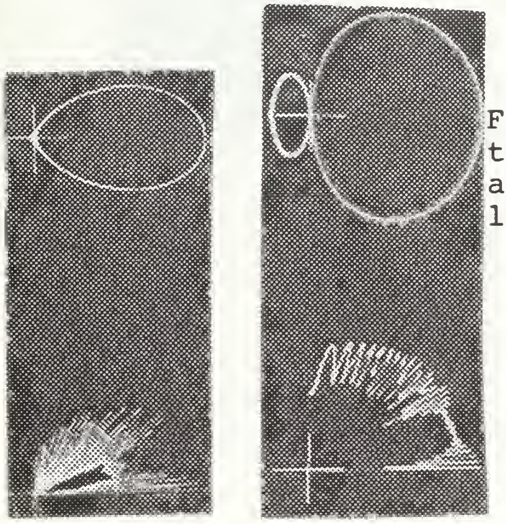


(a) Dry Land
G_p = 10.7 db
FBR_p = 9.6 db

(b) Fresh Water
G_p = 9.9 db
FBR_p = 9.5 db

(c) Sea Water
G_p = 12.7 db
FBR_p = 9.5 db

Figure 10. Ten MH three element vertical array radiating over three types of terrain.[17]



(a) Linear

(b) Log

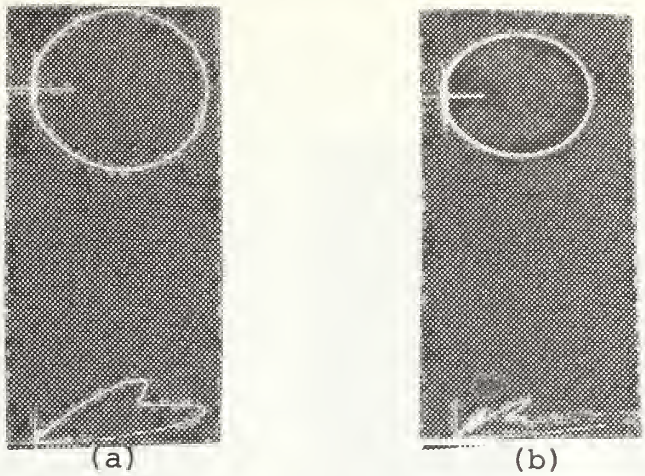
Figure 11. 150 MH Optimal three element horizontal array--linear plot (a), and log plot (b).[18]

G_p = 12.9 db
FBR_p = 22.0 db
R_{in} = 28.7 Ω
X_{in} =-13.5 Ω

---

[17] L_i=.506,.500,.450; d_i=.233,.133; h_t=2.; f_{MH}=10.;ε=10., $\sigma=10^{-3}$, ϕ_p=90., θ_p=84.(land); ε=80., σ=2., ϕ_p=90., θ_p=76.(H_2O); ε=80., $\sigma=10^{-2}$, ϕ_p=90., θ_p=75.(sea). (L, d and h_t are in λ)

[18] L_i=.510,.490,.460; d_i=.250,.08; h_t=15; ε=5.; $\sigma=10^{-3}$; ϕ_p=90.; θ_p=89. (L, d and h_t are in λ)

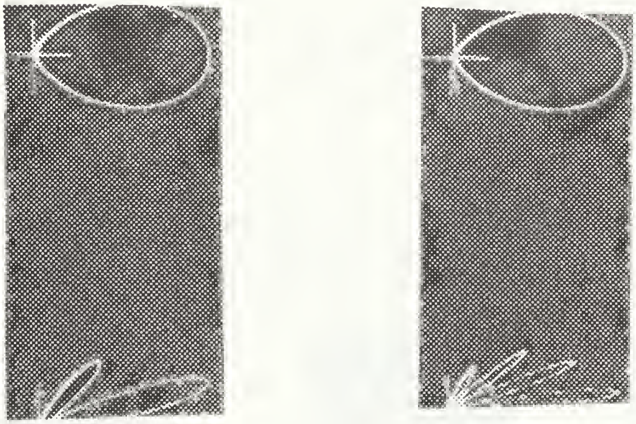


(a)

h_t = 1.0
G_p = 8.6 @ $_p$=80.0
FBR_p = 9.6

(b)

h_t = 2.0
G_p = 10.7 @ θ_p=84.0
FBR_p = 9.6



(c)

h_t = 1.0
G_p = 12.9 @ $_p$=76.0
FBR_p = 27.0

(d)

h_t = 2.0
G_p = 13.3 @ $_p$=83.
FBR_p = 27.3

Figure 12. Ten MHz 3-element vertical arrays (a) & (b) and horizontal arrays (c) & (d) at different height over land.[19] (G and FBR in db)

---

[19] See figure 16 for parameters. (h_t is in λ.)

(a)
ω = 0.
stable

(b)
ω = 30.
pt.roll = 15.4°
up pitch = 5.5°
θ' = 71.5
α = 64.0
h = 1.82

(c)
ω = 90.
st.roll = 31.0°
dn.pitch = 11.0°
θ' = 120.0
α = -11.0
h = 1.68

Figure 13. Ten MH three element horizontal array aboard ship in rough seas. Sea state = 7 and relative direction of waves = 40. Gain varies.[20]

(a)
ω = 0.
stable

(b)
ω = 30.
pt.roll = 21.0°
up pitch = 3.6°
θ' = 69.0
α = 3.6
h = 2.80

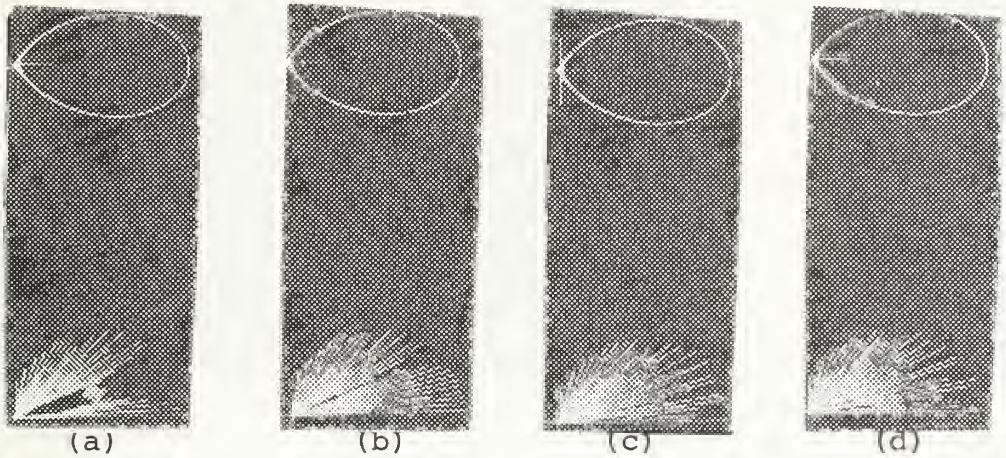
(c)
ω = 90.
pt.roll = 41.5°
up.pitch = 7.2°
θ' = 48.5
α = 7.2
h = 2.20

Figure 14. Thirty MH three element horizontal array aboard ship in rough seas. Sea state = 6 and relative direction of waves = 60. Gain varies.[21]

---

[20] L_i=.506,.500,.450; d_i=.233,.133; ε=80.; σ=5.; ϕ=90.; θ =80.; h_t=2. (L, d and h_t are in λ)

[21] L_i=.51,.50,.46; d_i=.25,.08; ε=80.; σ=5.; ϕ=90.; θ=85.; h_t=3. (L, d and h_t are in λ)



(a)
= 0.
stable

(b)
= 30.
15.4° p.roll
5.5° u.pitch
$\theta' = 58.5$
$\alpha = 5.5$
$h = 14.5$

(c)
= 90.
31.° p.roll
11.° u.pitch
$\theta' = 59.0$
α 11.0
$h = 12.7$

(d)
= 240.
27.° s.roll
9.6° d.pitch
$\theta' = 120.$
$\alpha = -9.6$
$h = 13.2$

Figure 15. 150 MHz 3-element horizontal array aboard ship in rough seas. Sea state = 6, and relative direction of waves = 40. Gain varies.[22]

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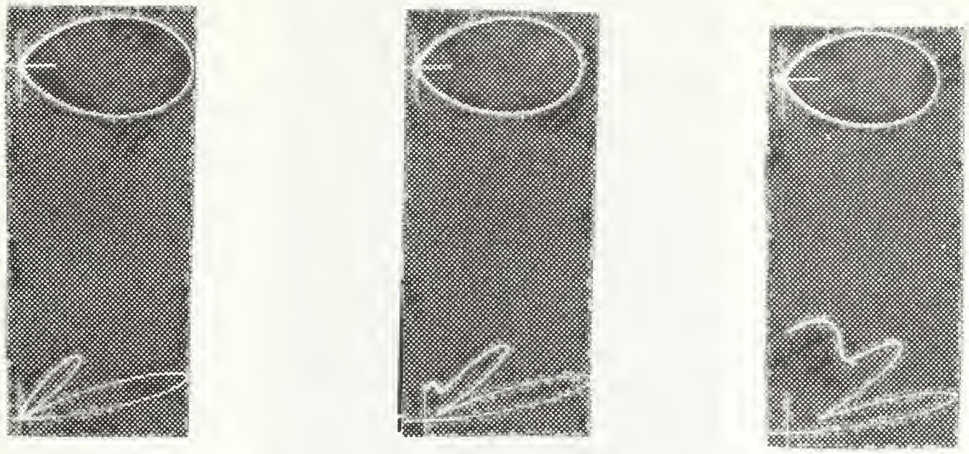
[22] Other parameters include L_i=.51,.50,.46; d_i=.25,.08; ε=80., σ=5., ϕ=90., θ=89., h=15.



(a)

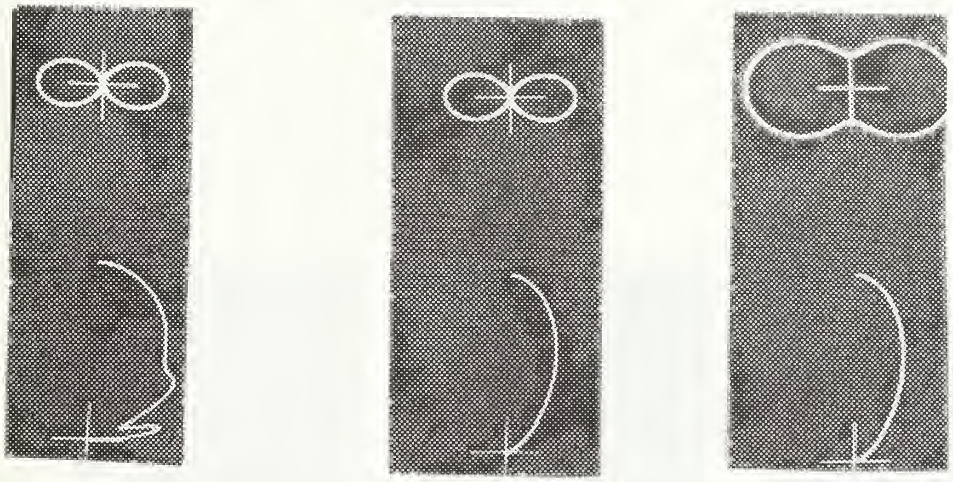
α = 0.0
G_p = 12.9@θ_p=76.0
FBR_p = 26.9

(b)

α = 30.0
G = 12.1@θ=76.0
G_p = 12.2@θ_p=78.0
FBR = 23.4

(c)

α = 60.0
G = 9.1@θ=76.0
G_p = 9.5@θ_p=79.0
FBR = 14.2



(d)

α = 90.0
G = 3.5@θ=76.0
G_p = 7.7@θ_p=1.0

(e)

α = 90.0
G = 13.3
FBR = 0.0
ϕ = 180.0
θ = 76.0

(f)

α = 90.0
G = 13.3
FBR = 0.0
ϕ = 0.0
θ = 50,

Figure 16. Ten MHz 3-element horizontal array at various tilt angles of array axis (bore sight).[23]

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[23] Various antenna parameters are: L_i=.506,.500,.450; d_i=.233,.133; ε=10., $\sigma=10^{-3}$, ϕ=90., θ=76. for (a)-(d), h_t=1.

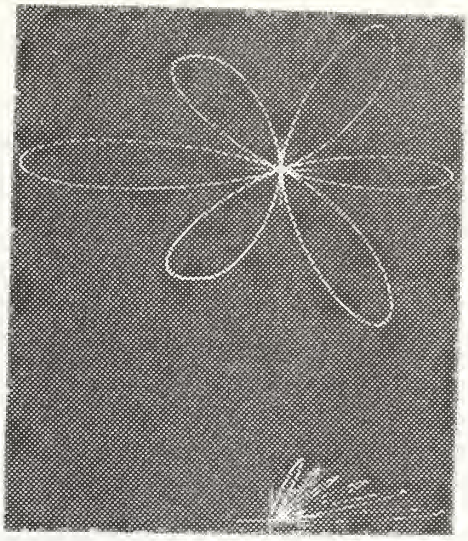
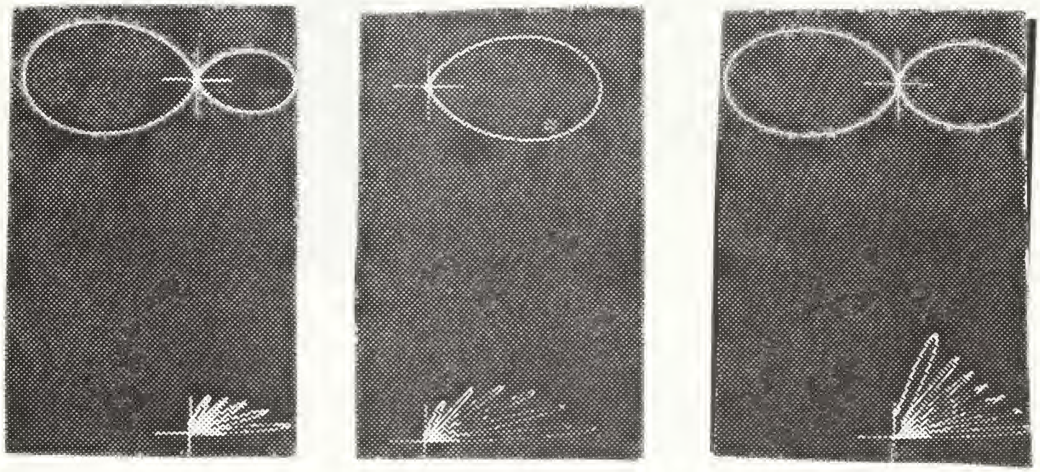


Figure 17. Ten MHz 3-element horizontal array operating over land at $f_{MH}=30$. (For antenna specifications see the ship-ocean model figure 14.)



(a) $f_{MH}=25$.

(b) $f_{MH}=30$.

(c) $f_{MH}=35$.

Figure 18. Thirty MHz 3-element horizontal array operating over land, above and below frequency for which array was designed. (For specifications, see the ship-ocean model figure 14.)

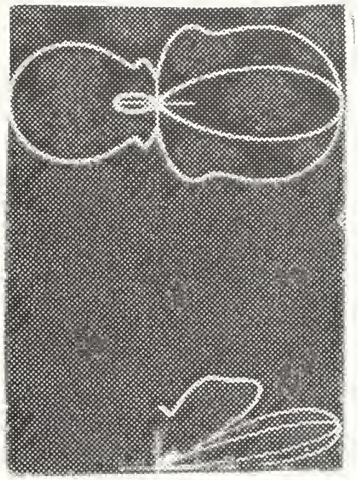


Figure 19. Ten MHz 5-element horizontal array over land. Comparison of linear(inter) plot with log plot. G_p=14.6 db, FBR_p=6.5 db.[24]

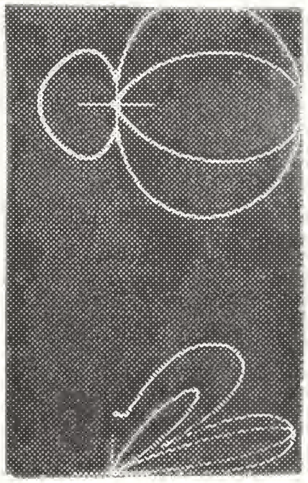


Figure 20. Ten MHz 3-element horizontal array over land. Comparison of linear (inter) plot with log plot. G_p=12.2 db, FBR_p=17.7 db.[25]

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[24] Various parameters include: L_i=.534,.500,.466,.466,.466; d_i=.25,.167,.167,.167; σ=10., $\sigma=10^{-2}$; ϕ=90.; θ=77.; h=1.

[25] Parameters for this array are: L_i=.550,.500,.433; d_i=.233,.133; all other parameters are the same as in footnote 24.

E. SUMMARY

There is a somewhat higher gain and an increasingly higher front-to-back ratio as the element spacing widens in the case of the optimal two element horizontal array possessing a reflecting parasite as compared to the optimal array with a director parasite.

The vertical multi-element array will produce a high front-to-back ratio--with values in the neighborhood of those of the horizontal array; however, if resistance is to be maintained above 20Ω it appears that the resulting optimal vertical array will not provide as high a front-to-back ratio as the optimal horizontal array. The optimal design of a multi-element horizontal array provides both a maximum front-to-back ratio and a near resonant input impedance, whereas the best design of a multi-element vertical array represents a compromise between these factors, e.g. a trade-off sacrifice exists.

The optimal design arrived at for a horizontal array becomes a suboptimal design when operated vertically, and vice versa.

Upon adding or eliminating directors from an array which has been designed for optimal performance--with the stipulation that parameters of all directors be kept uniform, the performance of the array will no longer be optimal.

Operation of the horizontal array over sea water environment provides a higher gain with respect to operation over land, but over fresh water provides a lower gain than over land.

Gain is considerably higher in the case of the vertical array and slightly higher with the horizontal array when they are radiating from a height of 2λ as compared with 1λ.

Gain varies considerably when the platform upon which an antenna operates is in heavy seas.

The gain of a horizontal array aimed vertically into the sky is lower than when it operates with no tilt angle whatsoever.

IV. RECOMMENDATIONS

At the beginning of this study the existing program which had been written for eight different single element antennas permitted the compilation of, at maximum, only three different antennas--the computer memory was exceeded when more options were added. Now the revised program allows all nine antennas to be compiled, with the eight unused antenna subroutines residing externally on drum. This permits consideration of further expanding the main program capabilities inasmuch as the effective computer memory has more capacity.

In regard to expanding the program's capability for solving array problems, it is recommended that the graphic input portion of the program be extended to allow for more element length and space parameters. This would permit study of long arrays (arrays having a cumulative spacing that total more than one wavelength).

It is recommended that an optimization routine be added to the main program which would encompass one or more of the following criteria: gain, front-to-back ratio, resonance of the driven element (Z_{22}), thickness of the element, and impedance (or standing wave ratio). For this purpose, a more powerful computer--which gives faster solution time--might be preferable.

APPENDIX A

DESCRIPTION OF PROGRAM OPERATING PROCEDURES

The program for the system of antennas as originally written has been changed to accommodate the Yagi-Uda. Accordingly, the program was rewritten in part to obtain more effective use of limited information storage in the computer. Redundancies between antenna programs have been reduced, the shipboard simulator and the gain pattern routine was rewritten to accommodate the Yagi-Uda and to save storage, and the graphic text segment was changed to satisfy the need for additional parameter inputs and outputs. The program remains functional for all antennas listed below. Figure 21 shows how the parameters and the output appear to the operator at the CRT graphics display console.

To operate the program, first ready the Graphics device to be used by following simple lab instructions for setting up "*Gated*." The program may be quickly loaded into the computer by mounting the program "*Dump*" tape on tape unit #1 and loading a few cards--BOOT,ΔJOB, ΔAGT, ΔASSIGN X1=MT1A, and ΔRERUN. Next, press the usual operating buttons on the 9300 console--IDLE, RESET, CLEAR FLAGS, RUN, and CARDS. The cards and tape are then read by the computer and within five seconds the computer console will print a request for the graphics device number that the user desires to interface with the computer. The user then responds by typing at the IDEV=1* and a carriage return. If a mistake is made in typing

start over again. If the message was successfully typed but for some reason the text did not appear on the graphics device, the quick load procedure may be used by pressing the following computer buttons: IDLE, RESET, STEP, RUN; then retype the message IDEV=1*. The graphics device should now contain the program text, providing all equipment is functional.

Each non-zero parameter entry must consist of a field-length of fewer than four numbers, plus a decimal, e.g. parameters should not exceed four characters of information. Upon completing a parameter entry, press the graphics TTY carriage-return and the blinking-light pointer will sequence to the location of the next parameter input. If a parameter is zero it need not be typed; press the carriage-return and go to the next entry location.[25] Correction to a typing mistake may be made only on the four-character field string of the parameter where the light-pointer is located. If the error is identified after sequencing to the next location the program may either be executed as is if no serious program error will result, of if the error can cause abort within the SDS-9300, either use the fast-restart[26] procedure or reload

---

[25] In addition to the need for a decimal to be entered with all data, a blanked-out space (one that has been skipped) is the same as a zero.

[26] This procedure is similar to the initial start-up: type the "Reset Gated" messages at the graphics teletype, and at the SDS-9300 press IDLE, RESET, STEP RUN and in response to the console message listed by the SDS-9300 type IDEV=1* (device #1).

the core "dump" tape again.[27] Either procedure takes less than 30 seconds whereas problem solution-time can take longer.

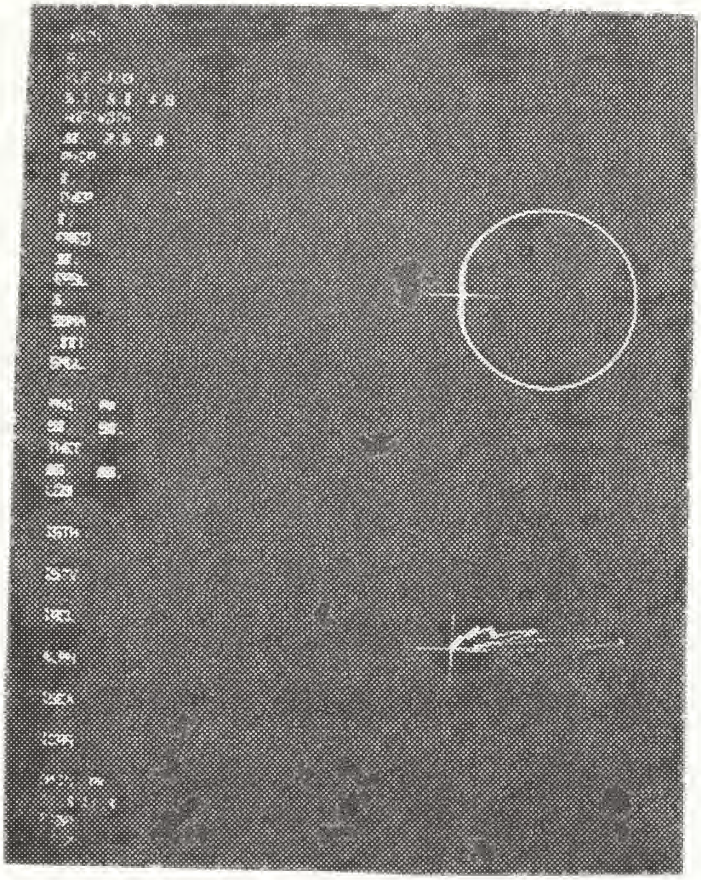


Figure 21. Photograph of graphic display showing parameter input/output.

---

[27] This procedure is identical to the initial set-up, in that the program dump tape is read into the computer. Press the button number "32" at the computer console, and type ΔRERUN at the console. This causes the dump tape to rewind and be read in again. Once again the computer should request Idev, and the user should respond by typing IDEV=1*.

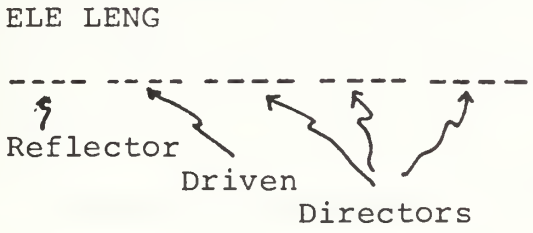
Observe the parameters shown in the photograph. The following is a description of these parameters to assist the user in specifying his antenna.

ANTN: Antenna

1. Tilted Dipole
2. Vertical Whip
3. Vertical Whip with Ground Screen
4. Inverted L
5. Sloping Long-wire
6. Sloping Vee
7. Horizontal Rhombic
8. Vertical Half-rhombic
9. Yagi-Uda

ELE LENG: Element length (meters)

If a single element antenna (ANTN 1 through ANTN 8) has been specified, the first available parameter space should be used for the element length and the other four spaces to the right left blank. If a Yagi-Uda (ANTN 9) has been chosen, the following order must be observed:

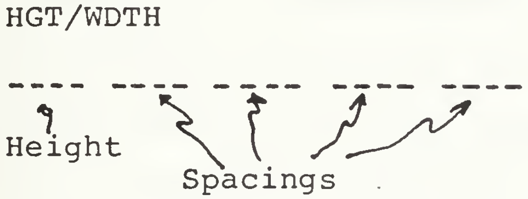


The user may choose to design the array without a reflector, in which case the first location at the left must be left blank. Any number of directors up to three may be selected,

however any blanked directors must be contiguous from the right. That is, skipping a director space then entering data into the next space, will result in the problem being misinterpreted: the first zero director marks the end of the array and determines the number of elements present.

<u>HGT/WDTH</u>: Height and width (meters)

A single element antenna requires only that the height be specified; the last four spaces may be left blank, since they will be ignored. If the Yagi-Uda is chosen, select the parameters according to the following format:



The first spacing is for the distance between the reflector and driven elements. Subsequent spacings are for the distance between respective directors.

<u>PHIP</u>: ϕ' Antenna parameter. Normally $\phi'=0^\circ$ will be used except under dynamic ship simulation, where the angle will specify the array axis relative to the ship's head. See figure 5.

<u>THEP</u>: θ' Antenna parameter. $\theta'=0^\circ$ for a vertical YAGI-UDA and $\theta'=90$ for horizontal.

<u>FREQ</u>: Frequency (MHz)

EPSL: Dielectric constant of earth

SGMA: Conductivity of earth (mho/m)

Typical values are:

Sea Water	Fresh Water	Wet Earth	Dry Earth
80	80	5-30	2-5
3-5	10^{-2}, 10^{-3}	10^{-1} - 10^{-3}	10^{-4}, 10^{-5}

SMUL: Multiplier

1. Multiplies σ by .1
2. Multiplies σ by .001

For numbers $<$ 3 this parameter may be used as a multiplier of the diameter of the array elements. The element diameter is used in determining the element spacing for computation of self impedance. When SMUL > 3 the expression used for spacing is

$$\frac{d}{\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{2}\ \ell_i}{200}, \text{ since } \quad r = \frac{\ell_i}{200}.$$

(see the computer program instruction that precedes #1925)

PHI: ϕ Observation parameter.

THET: θ Observation parameter.

Generally the observer wishes to position himself at the point of maximum gain, and take the azimuth and elevation sections there. The first solution of the problem will show where this point is located by listing the respective angles under PK: "Peak". The user may then enter these values for PHI and THET, and solve the problem again.

LOG: Log pattern

If left blank linear patterns will result. If "1." is entered, a previously entered pattern via light pen will be erased. If "2." is entered, a log pattern will result.

ISTH: Store the horizontal pattern

ISTV: Store the vertical pattern

Leave blank except to store the pattern about to be calculated for purposes of future comparison with another pattern by means of "double-exposure". Enter "1." to store.

IRCL: Recall a stored pattern

Left blank until patterns have been stored by a previous problem solution using ISTH/ISTV. When comparison is desired with the solution of the problem being entered, enter "1". Ensure that if recall is desired ISTH and ISTV are left blank, otherwise comparison will be made of the solution with itself.

ALPH: Array-axis angle

The tilt angle (or the angle of elevation with the ground) of the Yagi-Uda array axis. This is the slope angle with the ground for the two sloping antennas (ANTN 5 and 6) as well as for the vertical half-rhombic (ANTN 8), and the minor angle between elements of the horizontal rhombic (ANTN 7).

ISEA: Sea state

For the tilted dipole, vertical whip, sloping longwire, and Yagi-Uda. Left blank unless iterative solutions under dynamic sea conditions are desired. The program is written for a sinusoidally varying sea with solutions produced at thirty degree intervals. The peak amplitude of roll is 8 degrees per sea state, and the peak pitch is 3 degrees per sea state.

ICRS: Course of the seas

Enter the direction of the seas relative to the ship's heading.

GAIN: Gain at the observation points specified by ϕ, θ.

The peak gain, identified under "PK" is also given. The ϕ and θ which locate the peak gain are listed adjacent to the values specified for the problem (discussed above under PHI and THET).

FTBR: Front-to-back ratio

The ratio of the peak forward-azimuth to peak reverse-azimuth gains. (This is not a comparison between the 90 and 270 degree gain values). See footnote 3.

COMPUTER PROGRAM

```
C---------MAIN-----CATHODE RAY TUBE SOLUTION OF ANTENNA PATTERNS
      COMMON   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1COSDL,COSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISOL,K,KAY,KOS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VOLTS,VOLDRI,WIRE,W0SQ,WYE,XGRAL,Y0,Z,Z0,ZZPAK
      REAL K,L,LMDA,LH,LHP,LHS,KCOS,NORM,KLTP,NORMR,NORMF
      REAL KOS
      INTEGER VPAT,PATRN,PAR,ANTN
      COMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      COMPLEX AJ,C2,ADA1,ADA2,FIFA,Y
      DIMENSION X(360),Y(360)
      DIMENSION IGDIR(6),PATRN(362),VPAT(92),X1(50),Y1(50),X2(90),
     1Y2(90),X3(360),Y3(360),IMD(362),Z(5,5),ISAVV(92),ISAVH(362),
     1ITRY(50),IM(50)
      DIMENSION LH(5 ),D(4),ITDIR(53),LABL(24),LNL(15),LND(28),IP(26)
      DIMENSION VOLTS(10),CRNT(10),IPAR(31),CUR(10),WYE(5),ZZPAC(10,10),
     1ZZPAK(100)
      DIMENSION GV(90),GH(360),FAC1(180),FAC2(180)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VOLTS,CRNT)
     1,(ZZPAC,ZZPAK)
      DATA(IPAR(I),I=1,28)/28*77777777B/
      DATA NULL/77777777B/
      DATA LNL/7,9,11,13,15,17,19,21,23,25,27,29,31,33,35/
      DATA LND/2,4,4,4,4,4,6,6,6,6,6,8,10,12,14,16,18,20,22,24,26,28,
     130,32,34,36,38,40/
      DATA IP/1,1,6,11,16,21,1,6,11,16,21,1,1,1,1,1,1,1,1,1,1,1,1,1,1/
      DATA VOLTS/0.,1.,0.,0.,0.,0.,0.,0.,0.,0./
      DATA LABL/4HANTN,4HELE ,4HLENG,4HHGT/,4HWDTH,4HPHIP,4HTHEP,
     14HFREQ,4HEPSL,4HSGMA,4HSMUL,4HPHI ,4H  PK,4HTHET,4HLOG ,4HISTH,4HI
     2STV,4HIRCL,4HALPH,4HISEA,4HICRS,4HGAIN,4H  PK,4HFTBR/
C     PARAMETER FORMAT PROCESSOR
      ISIZE=6
      NAMELIST IDEV,IER
      OUTPUT(101)'TYPE IDEV'
```

```
      INPUT(101)
      CALL DTINIT(IDEV,ITDIR,53,IER)
      IF(IER.NE.0)ØUTPUT(101)IER,'DTINIT'
1     CØNTINUE
      DØ 2   I=1,50
2     IM(I)=0
      DØ 3   I=1,50
3     X1(I)=Y1(I)=0.0
      DØ 4   I=1,90
4     X2(I)=Y2(I)=0.0
      DØ 5   I=1,360
5     X3(I)=Y3(I)=0.0
6     CØNTINUE
      CALL TEXTØ(IDEV,LABL(1),1,1,1,1,3,IER)
      CALL TEXTØ (IDEV,LABL(2),2,3,1,1,3,IER)
      CALL TEXTØ (IDEV,LABL(4),2,5,1,1,3,IER)
      DØ 7  I=1,6
      CALL TEXTØ (IDEV,LABL(I+5),1,LNL(I),1,1,3,IER)
7     IF(IER.NE.0)ØUTPUT(101)IER,'TBLK',I
      CALL TEXTØ(IDEV,LABL(12),2,LNL(7),1,1,3,IER)
      DØ 8   I=8,15
      CALL TEXTØ (IDEV,LABL(I+6),1,LNL(I),1,1,3,IER)
8     IF (IER.NE.0)ØUTPUT(101)IER,'TBLK',I
      CALL TEXTØ (IDEV,LABL(22),2,37,1,1,3,IER)
      CALL TEXTØ (IDEV,LABL(24),1,39,1,1,3,IER)
      IF(IER.NE.0)ØUTPUT(101)IER,'TBLK',I
C    PARAMETER AND ØPTIØNS CØMMAND INPUT PRØCESSØR
      DØ 10 I=1,26
      CALL TEXTR(IDEV,IPAR(I),1,LND(I),IP(I),1,3,IER)
      IF(IER.NE.0)ØUTPUT(101)IFR,'TBLK',I
9     IF(MØD(ITDIR(I+20),8).EQ.0) GØ TØ 9
      CALL TEXTI(IDEV,IPAR(I),1,LND(I),IP(I),IER)
10    IF(IER.NE.0)ØUTPUT(101)IER,'IPAR',I
      IPAR(27)=IPAR(28)=IPAR(29)=IPAR(30)=IPAR(31)=-1
      CALL TEXTØ(IDEV,IPAR(27),1,38,1,1,3,IER)
```

```
      CALL TEXT0(IDEV,IPAR(28),1,38,6,1,3,IER)
      CALL TEXT0(IDEV,IPAR(29),1,40,1,1,3,IER)
      CALL TEXT0(IDEV,IPAR(30),1,20,6,1,3,IER)
      CALL TEXT0(IDEV,IPAR(31),1,22,6,1,3,IER)
      DECODE (4,15, IPAR(1)) ANTN
      DECODE (4, 12,IPAR(2)) LH(1)
      DECODE (4, 12,IPAR(3)) LH(2)
      DECODE (4, 12,IPAR(4)) LH(3)
      DECODE (4, 12,IPAR(5)) LH(4)
      DECODE (4, 12,IPAR(6)) LH(5)
      DECODE (4, 12,IPAR(7)) H
      DECODE (4, 12,IPAR(8)) D(2)
      DECODE (4, 12,IPAR(9)) D(3)
      DECODE (4, 12,IPAR(10)) D(4)
      DECODE (4, 12,IPAR(11)) D(5)
      DECODE (4,15, IPAR(12)) ITEM
      PHIPR=ITEM
      DECODE (4,15, IPAR(13)) ITEM
      THEPR=ITEM
      DECODE (4, 11,IPAR(14)) F
      DECODE (4, 12,IPAR(15)) EPSLN
      DECODE (4, 13,IPAR(16)) SIGMA
      DECODE (4, 12, IPAR(17)) SMUL
      DECODE (4,15, IPAR(18)) M
      DECODE (4,15, IPAR(19)) KAY
      DECODE (4,15, IPAR(20)) PAR
      DECODE (4,15, IPAR(21)) ISTRH
      DECODE (4,15, IPAR(22)) ISTRV
      DECODE (4,15, IPAR(23)) IRCAL
      DECODE (4,15, IPAR(24)) ITEM
      ALPH=ITEM
      DECODE (4,15, IPAR(25)) ISEA
      DECODE (4,15, IPAR(26)) ICRS
      L=LH(1)
11    FORMAT(F4.0)
```

```
12    FØRMAT(F4.1)
13    FØRMAT(F4.2)
14    FØRMAT(F4.3)
15    FØRMAT(I4)
      IF(SMUL.EQ.(1.))SIGMA=SIGMA*.1
      IF(SMUL.FQ.(2.))SIGMA=SIGMA*.01
      IF (ANTN.LT.9)GØ TØ 155
      T=1.
      IF (SMUL.GT.2.) T=SMUL
155   WRITE(6,16  ) ANTN,(LH(I),I=1,5),H,(D(I),I=2,5),PHIPR,THEPR,F,
     1EPSLN,SIGMA,      M,KAY,PAR,ISTRH,ISTRV,IRCAL,ALPH,ISEA,ICRS,T
16    FØRMAT(1H1,$ANTN=$,I2,/,$LH(I)=$,5F10.2,/,$H=$,F5.1,/,$D(I)=$,
     14F13.3,/,$PHIPR=$,F5.1,/,$THEPR=$,F5.1,/,$F=$,F7.3,/,$EPSLN=$,
     2F4.1,/,$SIGMA=$,F9.5,/,$M(PHI)=$,F5.1,/,$KAY(THET)=$,F5.1,/,$PAR=$
     3,I2,/,$ISTRH=$,I2,/,$ISTRV=$,I2,/,$IRCAL=$,I2,/,$ALPH=$,F5.1,/,
     4$ISEA=$,I2,/,$ICRS=$,I2,/,$T (MULT FØR DIAM-LENGTH)=$,F7.3,/,/)
      NE=0
      IF (ANTN.NE.9) GØ TØ 20
      DØ 17 I=1,5
      LH(I)=LH(I)/2
17    CØNTINUE
      IF (LH(2).EQ.0.0) GØ TØ 19
      IF (LH(1).EQ.0.0) GØ TØ 18
      IF((LH(3).EQ.0.0).AND.(LH(4).EQ.0.0).AND.(LH(5).EQ.0.0)) NE=2
     1; GØ TØ 20
      IF((LH(4).EQ.0.0).AND.(LH(5).EQ.0.)) NE=3; GØ TØ 20
      IF((LH(5).EQ.0.0)) NE=4; GØ TØ 20
      IF ((LH(2).NE.0.0).AND.(LH(3).NE.0.0).AND.(LH(4).NE.0.0).AND.
     1 (LH(5).NE.0.0)) NE=5; GØ TØ 20
      GØ TØ 19
18    IF((LH(3).EQ.0.0).AND.(LH(4).EQ.0.0).AND.(LH(5).EQ.0.0)) NE=1
     1; GØ TØ 20
      IF((LH(4).EQ.0.0).AND.(LH(5).EQ.0.)) NE=2; GØ TØ 20
      IF((LH(5).FQ.0.0)) NF=3; GØ TØ 20
      IF ((LH(2).NE.0.0).AND.(LH(3).NE.0.0).AND.(LH(4).NE.0.0).AND.
```

```
     1 (LH(5).NE.0.0)) NE=4; GØ TØ 20
19     IF (NE.EQ.0) ØUTPUT(101) 'ILLEGAL ZERØ LENGTHED ELEMENT. START
     1ØVER'; GØ TØ 6
20     IF(PAR.EQ.1) GØ TØ 1
       CALL DGINIT(IDEV,IGDIR,ISIZE,IER)
       IF(PAR.EQ.1)GØ TØ 1
C      PATTERN MANUAL ENTRY PRØCESSØR
       DØ 21  I=1,50
21     IMD(I)=IM(I)
       ITRY(1)=IHEAD(0,10)
       ITRY(2)=IPACK(0,.6,0)
       ITRY(3)=IPACK(0,.4,1)
       ITRY(4)=IPACK(-.1,.5,0)
       ITRY(5)=IPACK(.1,.5,1)
       ITRY(6)=IPACK(0,-.6,0)
       ITRY(7)=IPACK(0,-.4,1)
       ITRY(8)=IPACK(-.1,-.5,0)
       ITRY(9)=IPACK(.1,-.5,1)
       ITRY(10)=IPACK(0,0,0)
       DØ 22  I=11,50
       J=I-1
22     ITRY(I)=IPACK(X1(I),Y1(I),IMD(I))
       CALL GRAPHR(IDEV,ITRY,50,1,IER)
       IF(IER.NE.0)ØUTPUT(101) IER,'GBLK1'
23     IF(MØD(IGDIR(1),8).EQ.0)GØ TØ 23
       CALL GRAPHI(IDEV,ITRY,1,IER)
       IF(IER.NE.0)ØUTPUT(101)IER,'IGBLK'
       DØ 24  I=1,50
       CALL UNPACK(ITRY(I),X1(I),Y1(I),IMD(I))
24     IM(I)=IMD(I)
C      ENVØRØNMENTAL CØNSTANTS PRØCESSØR
       PHIPR=PHIPR*(3.14159265/180)
       THEPR=THEPR*(3.14159265/180)
       ALPH=(3.14159265/180)*ALPH
       ALPCM=(3.14159265/2.0)-ALPH
```

```
      DLPRI=(3.14159265/2.0)-THEPR
      F=F*1.0E 06
      AOMEG=2*3.14159265*F
      IF(ANTN.NE.3) GO TO 26
      ADA1=CMPLX(0.,1.26E-06*AOMEG)
      ADA2=CMPLX(SIGMA,AOMEG*EPSLN*8.854E-12)
      ADA=(ADA1/ADA2)**.5
      TEMP1=REAL(ADA)
      TEMP2=AIMAG(ADA)
      WRITE(6,25  )TEMP1,TEMP2
25    FORMAT('ADA=',2F12.1)
26    CONTINUE
      LMDA=3.0E08/F
      K=6.28318530/LMDA
      F=F*1.0E-06
      C2=K*(CMPLX(EPSLN,-1.8E04*SIGMA/F))**0.5
      HTEMP=H
      THTEM=THEPR
      ALTEM=ALPH
      RHPRI=(K-C2)/(K+C2)
      RVPRI=(C2-K)/(C2+K)
      II=0
      DPHIP=0.0
C-----ENTRY IS BEING MADE INTO THE SHIP DYNAMICS LOOP,AND INTO THE PATTE
C-----GAIN LOOP. BEYOND THIS POINT ANTENNA ORIENTATION PARAMETERS  MAY C
C-----AND THE OBSERVATION ANGLE PARAMETERS WILL CHANGE.
      IF(ISEA.GT.0)GO TO 80
27    CONTINUE
      ISOL=0
C     INPUT RESISTANCE PROCESSOR
      IF (ANTN.EQ.1) CALL BRANCH1
      IF (ANTN.EQ.2) CALL BRANCH2
      IF (ANTN.EQ.3) CALL BRANCH3
      IF (ANTN.EQ.4) CALL BRANCH4
      IF (ANTN.EQ.5) CALL BRANCH5
```

```
      IF (ANTN.EQ.6) CALL BRANCH6
      IF (ANTN.EQ.7) CALL BRANCH7
      IF (ANTN.EQ.8) CALL BRANCH8
      IF (ANTN.EQ.9) CALL BRANCH9
28    CONTINUE
      IF(ISEA.GT.0)GO TO 29
      WRITE (6,30 )RIN
29    CONTINUE
30    FORMAT ($RIN=$,F12.1)
C     OBSERVATION ANGLE CONSTANTS PROCESSOR
      DO 34 N=1,2
      IF(N.EQ.1) GO TO 31
      IF(N.EQ.2) GO TO 32
31    DO 34 I=1,90
      J=M
      GO TO 33
32    DO 34 J=1,360
      I=KAY
      GO TO 33
33    CONTINUE
      THETA=I*(3.14159265/180)
      DELTA=3.14159264/2.-THETA
      PHI=J*(3.14159265/180)
      KOS=COS(THETA)*COS(THEPR)+SIN(THETA)*SIN(THEPR)*COS(PHI-PHIPR)
      SINSQ=1-(KOS**2)
      WOSQ=(3.14159265/180)**2
      IF(SINSQ.LT.WOSQ)SINSQ=WOSQ
      FIFA=(1-((K/C2)*SIN(THETA))**2)**0.5
      KCOS=COS(THETA)
      RV=(KCOS-(K/C2)*FIFA)/(KCOS+(K/C2)*FIFA)
      RH=(KCOS-(C2/K)*FIFA)/(KCOS+(C2/K)*FIFA)
      CV=CABS(RV)
      CH=CABS(RH)
      VR=REAL(RV)
      VI=AIMAG(RV)
```

```
      SIGHV=ATAN2(VI,VR)
      HR=REAL(RH)
      HI=AIMAG(RH)
      SIGHH=ATAN2(HI,HR)
      S1=CØS(SIGHH-2*K*H*CØS(THETA))
      S2=SIN(SIGHH-2*K*H*CØS(THETA))
      S3=CØS(SIGHV-2*K*H*CØS(THETA))
      S4=SIN(SIGHV-2*K*H*CØS(THETA))
      SINDL=SIN(DELTA)
      CØSDL=CØS(DELTA)
      SINDP=SIN(DLPRI)
      CØSDP=CØS(DLPRI)
      AJ=CMPLX(0,1)
      ØNE=CØS(DLPRI)
      TWØ=-SIN(DLPRI)
      CEE=(RHPRI*CØS(DLPRI)+AJ*RVPRI*SIN(DLPRI))*CMPLX(ØNE,TWØ)
      IF (ANTN.EQ.1 ) CALL BRANCH1A
      IF (ANTN.EQ.2 ) CALL BRANCH2A
      IF (ANTN.EQ.3 ) CALL BRANCH3A
      IF (ANTN.EQ.4 ) CALL BRANCH4A
      IF (ANTN.EQ.5 ) CALL BRANCH5A
      IF (ANTN.EQ.6 ) CALL BRANCH6A
      IF (ANTN.EQ.7 ) CALL BRANCH7A
      IF (ANTN.EQ.8 ) CALL BRANCH8A
      IF (ANTN.EQ.9 ) CALL BRANCH9A
34    CØNTINUE
C     NØRMALIZE AND MAX GAIN PRØCESSØR
      NØRM=NØRMF=NØRMR=GVER=GHØR=0.0
      DØ 35 I=1,360
      IF((I.LE.90).AND.(GVER.LT.GV(I)))GVER=GV(I);IV=I
      IF                (GHØR.LT.GH(I))GHØR=GH(I);IH=I
      IF(GVER.GT.GHØR)NØRM=GVER
      IF(GHØR.GT.GVER)NØRM=GHØR
      IF ((ANTN.NE.9).ØR.((ANTN.EQ.9).AND.(I.GT.180))) GØ TØ 35
      NØRMF=AMAX1(NØRMF,FAC1(I))
```

```
      NØRMR=AMAX1 (NØRMR,FAC2(I))
35    CØNTINUE
      FBR=NØRMF/NØRMR
      FBR=10*ALØG10(FBR)
      GMAX=NØRM/RIN
      GMAX=10*ALØG10(GMAX)
      GVER=10*ALØG10(GVER/RIN)
      GHØR=10*ALØG10(GHØR/RIN)
      GAIN=10*ALØG10(GV(KAY)/RIN)
      ENCØDE(4,12,IPAR(27))GAIN
      ENCØDE(4,12,IPAR(28))GMAX
      ENCØDE(4,12,IPAR(29))FBR
      CALL TEXTØ(IDEV,IPAR(27),1,38,1,1,3,IER)
      CALL TEXTØ(IDEV,IPAR(28),1,38,6,1,3,IER)
      CALL TEXTØ(IDEV,IPAR(29),1,40,1,1,3,IER)
      IF(IER.NE.0)ØUTPUT(101)IER,'GAIN'
      IF (PAR.EQ.2) GØ TØ 62
36    CØNTINUE
      WRITE(6,37 ) GAIN,GMAX,GVER,IV,GHØR,IH
37    FØRMAT(1H0,$GAIN=$,F6.2,$DB$,/,$GMAX=$,F6.2,$DB$,10X,$MAX GAIN VER
     1T=$,F6.2,$DB$,2X,$THETA=$,I2,10X,$MAX GAIN HØR=$,F6.2,$DB$,2X,$PHI
     2=$,I2)
      IF (ANTN.EQ. 9) WRITE (6,38 ) FBR
38    FØRMAT ($F/B RATIØ=$,F5.1,$DB$)
      ENCØDE(4,11,IPAR(30))IH
      ENCØDE(4,11,IPAR(31))IV
      CALL TEXTØ(IDEV,IPAR(30),1,20,6,1,3,IER)
      CALL TEXTØ(IDEV,IPAR(31),1,22,6,1,3,IER)
39    CØNTINUE
C     PATTERN DISPLAY PRØCESSØR
      DØ 40 I=1,360
      PHI=I*(3.14159265/180)
      GH(I)=GH(I)/(NØRM*2.)
      X(I)=GH(I)*SIN(PHI)
40    Y(I)=GH(I)*CØS(PHI)+0.5
```

```
      IMD(1)=0
      DO 41 I=2,360
41    IMD(I)=1
      PATRN(1)=IHEAD(0,10)
      DO 42 I=2,361
      J=I-1
42    PATRN(I)=IPACK(X(J),Y(J),IMD(J))
      PATRN(362)=0
      IF(ISEA.GT.0)GO TO 84
      CALL GRAPHR(IDEV,PATRN,362,2,IER)
      IF(IER.NE.0)OUTPUT(101)IER,'GBLK'
43    IF(MOD(IGDIR(2),8).EQ.0)GO TO 43
44    CONTINUE
      IF(ISTRH.EQ.1) GO TO 52
45    CONTINUE
C     DISPLAY VERT PATTERN AT REQUESTED PHI
      DO 46 I=1,90
      THETA=I*(3.14159265/180)
      GV(I)=GV(I)/(NORM*2.)
      X(I)=GV(I)*SIN(THETA)
46    Y(I)=GV(I)*COS(THETA)-.5
      IMD(1)=0
      DO 47 I=2,90
47    IMD(I)=1
      VPAT(1)=IHEAD(0,10)
      DO 48 I=2,91
      J=I-1
48    VPAT(I)=IPACK(X(J),Y(J),IMD(J))
      VPAT(92)=0
      IF(ISEA.GT.0)GO TO 85
      CALL GRAPHR(IDEV,VPAT,92,3,IER)
      IF(IER.NE.0) OUTPUT(101)IER,'GBLK2'
49    IF(MOD(IGDIR(3),8).EQ.0)GO TO 49
50    CONTINUE
      IF(ISTRV.EQ.1)GO TO 54
```

```
51      CØNTINUE
        IF(IRCAL.EQ.1)GØ TØ 56
        IF(ISEA.GT.0)GØ TØ 80
        GØ TØ 6
C       PATTERN SAVE PRØCESSØR
52      CALL GRAPHI(IDEV,PATRN,2,IER)
        IF(IER.NE.0)ØUTPUT(101)IER,'GBLK2'
        DØ 53  I=1,360
53      CALL UNPACK(PATRN(I+1),X3(I),Y3(I),IMD(I))
        GØ TØ 45
54      CALL GRAPHI(IDEV,VPAT,3,IER)
        IF(IER.NE.0)ØUTPUT(101)IER,'GBLK3'
        DØ 55  I=1,90
55      CALL UNPACK(VPAT(I+1),X2(I),Y2(I),IMD(I))
        GØ TØ 51
C       DISPLAY SAVED PATTERNS PRØCESSØR
56      IMD(1)=0
        ISAVH(1)=IHEAD(0,10)
        DØ 57  I=2,360
57      IMD(I)=1
        DØ 58  I=2,361
        J=I-1
58      ISAVH(I)=IPACK(X3(J),Y3(J),IMD(J))
        ISAVH(362)=0
        CALL GRAPHR(IDEV,ISAVH,362,4,IER)
        IF(IER.NE.0)ØUTPUT(101)IER,'GBLK4'
59      IF(MØD(IGDIR(4),8).EQ.0)GØ TØ 59
        ISAVV(1)=IHEAD(0,10)
        DØ 60  I=2,91
        J=I-1
60      ISAVV(I)=IPACK(X2(J),Y2(J),IMD(J))
        ISAVV(92)=0
        CALL GRAPHR(IDEV,ISAVV,92,5,IER)
        IF(IER.NE.0)ØUTPUT(101)IER,'GBLK5'
61      IF(MØD(IGDIR(5),8).EQ.0)GØ TØ 61
```

```
      GØ TØ 6
C     LØG GAIN PRØCESSØR
62    BLIM=.001
      DØ 63  I=1,90
      TEMP=GV(I)/NØRM
      IF(TEMP.LT.BLIM)TEMP=BLIM
      GV(I)=ALØG10(TEMP)+3.
63    CØNTINUE
      DØ 64  I=1,360
      TEMP=GH(I)/NØRM
      IF(TEMP.LT.BLIM)TEMP=BLIM
      GH(I)=ALØG10(TEMP)+3.
64    CØNTINUE
      NØRM=3.
      GØ TØ 36
80    II=II+3
      PI=3.14159265
      D2R=PI/180.0
      VAR=(PI/18.0)*II
      WAVE=(ISEA*8*SIN(VAR))*D2R
      VAR1=ICRS*D2R
      DLT1=WAVE*SIN(VAR1)
      DLT2=WAVE*CØS(VAR1)*0.3
      IF(ANTN.EQ.5)GØ TØ 81
      H=HTEMP*CØS(DLT1)*CØS(DLT2)
      IF(ANTN.EQ.9) GØ TØ 86
      THEPR=THTEM-DLT1
      DLPRI=PI/2.-THEPR
      GØ TØ 82
81    AA=2*L*SIN(DLT1/2.)
      BB=2*L*SIN(DLT2/2.)
      CC=SQRT(AA**2+BB**2)
      DD=SQRT(L**2-(CC/2.)**2)
      DLT3=2.*ATAN2((CC/2.),DD)
      SIND3=SIN(DLT3)
```

```
      IF((SIND3.LT.WOSQ).AND.(SIND3.GE.0.0))SIND3=WOSQ
      IF((SIND3.GT.-WOSQ).AND.(SIND3.LT.0.0))SIND3=-WOSQ
      SINA=SIN(DLT1)/SIND3
      SINA=ABS(SINA)
      CØSA=SQRT(1.0-SINA**2)
      DPHIP=ATAN2(SINA,CØSA)
      IF((DLT1.LT.0.0).AND.(DLT2.GE.0.0))DPHIP=-DPHIP
      IF((DLT1.LT.0.0).AND.(DLT2.LT.0.0))DPHIP=-(PI-DPHIP)
      IF((DLT1.GT.0.0).AND.(DLT2.LT.0.0))DPHIP=PI-DPHIP
      THEPR=THTEM+DLT3
      DLPRI=PI/2.-THEPR
82    IF(II.EQ.39)GØ TØ 6
      WRITE(6,83) WAVE,H,ALPH,THEPR,II
83    FØRMAT(1HO,$WAVE=$,F7.3,$H=$,F7.3,$ALPH=$,F7.3,$THEPR=$,F7.3,$ØMEG
     1A=$,F7.3)
      GØ TØ 27
84    CALL GRAPHØ(IDEV,PATRN,362,2,IER)
      IF(IER.NE.0)ØUTPUT(101)IER,'HPAT'
      GØ TØ 44
85    CALL GRAPHØ(IDEV,VPAT,92,3,IER)
      IF(IER.NE.0)ØUTPUT(101)IER,'VPAT'
      GØ TØ 50
86    THEPR=THTEM-(DLT1*CØS(PHIPR)+DLT2*SIN(PHIPR))
      ALPH=ALTEM+DLT2*CØS(PHIPR)-DLT1*SIN(PHIPR)
      DLPRI=PI/2.-THEPR
      GØ TØ 82
      END
```

```
C---------ANTN 1
C-----ARBITRARILY TILTED DIPØLE
      SUBRØUTINE BRANCH1
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT),
     1(ZZPAC,ZZPAK)
 1100 LHS=LHP=L/2
      CUMDIS=.00001
      ISØL=1
      DØ 1110 I=1,2
      IF(I.EQ.2) CUMDIS=2*H;ISØL=0
      CALL ZINT
 1110 Z(I,1)=CMPLX(RGRAL,XGRAL)
      RIN=REAL(Z(1,1))+REAL(Z(2,1)*CEE)
      RETURN
      END
```

```
      SUBROUTINE BRANCH1A
C--------ANTN 1
      COMMON   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1COSDL,COSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISOL,K,KAY,KOS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VOLTS,VOLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KOS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      COMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSION CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VOLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VOLTS,CRNT)
     1,(ZZPAC,ZZPAK)
  100 SINPI=SIN(PHI-PHIPR)
      COSPI=COS(PHI-PHIPR)
      FCT=1.0-(SINDL*SINDP+COSDL*COSDP*SINPI)**2
      FCTR=1.0-(-SINDP*SINDL+COSDL*COSDP*SINPI)**2
      GI=(COS(0.5*K*L*(SINDL*SINDP+COSDL*COSDP*SINPI))-COS(0.5*K*L))/FCT
      DI=(COS(0.5*K*L*(COSDL*COSDP*SINPI-SINDL*SINDP))-COS(0.5*K*L))/
     1FCTR
      ETHT1=(COSDP*SINPI*SINDL-SINDP*COSDL)*GI-(COSDP*SINPI*SINDL+
     1SINDP*COSDL)*DI*CV*S3
      EPHI1=COSDP*COSPI*(GI+DI*CH*S1)
      ETHT2=(COSDP*SINPI*SINDL+SINDP*COSDL)*DI*CV*S4
      EPHI2=COSDP*COSPI*DI*CH*S2
      IF(FCT.LT.WOSQ)ETHT1=EPHI1=0.0
      IF(FCTR.LT.WOSQ)ETHT2=EPHI2=0.0
      G=120.*(ETHT1**2+ETHT2**2+EPHI1**2+EPHI2**2)
      IF(N.EQ.1) GV(I)=G
      IF(N.EQ.2) GH(J)=G
      RETURN
      END
```

```
C--------ANTN 1
C-----REQUIRED FØR DIPØLE AND YAGI UDA
      SUBRØUTINE ZINT
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
      IF (ISØL.EQ.0) GØ TØ 1940
      SVDLP=DLPRI
      DLPRI=0.
1940  YO=CUMDIS*CØS(DLPRI)/LMDA
      ZO=-CUMDIS*SIN(DLPRI)/LMDA
      S=-LHS/LMDA
      RGRAL=.5*RESIST(S)
      DS=LHS/(50.*LMDA)
      DØ 1950 N=2,100
      S=S+DS
1950  RGRAL=RGRAL+RESIST(S)
      RGRAL=-30.*(RGRAL+.5*RESIST(LHS/LMDA))*DS
      S=-LHS/LMDA
      XGRAL=.5*REACT(S)
      DØ 1960 N=2,100
      S=S+DS
1960  XGRAL=XGRAL+REACT(S)
      XGRAL=-30.*(XGRAL+.5*REACT(LHS/LMDA))*DS
      IF (ISØL.EQ.1) DLPRI=SVDLP
      RETURN
      END
```

```
C--------ANTN 1
       FUNCTIØN RESIST(S)
       CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
      1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
      2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
      3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
      4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,7,ZO,ZZPAK
       REAL K,KØS,L,LH,LHP,LHS,LMDA
       INTEGER ANTN,PAR
       CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
       DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
      1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
       EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
      1,(ZZPAC,ZZPAK)
       IF (ANTN.LT.9) LS=LP=L
       IF (ANTN.GE.9) LP=2*LHP;LS=2*LHS
       SZ=S*CØS(2*DLPRI)
       SY=-S*SIN(2*DLPRI)
       TERM=YO+SY
       RØW2=(YO+SY)**2
       CA=ZO+SZ
       CA1=CA+0.5*LP/LMDA
       CA2=CA-0.5*LP/LMDA
       R=SQRT(RØW2+CA**2)
       R1=SQRT(RØW2+CA1**2)
       R2=SQRT(RØW2+CA2**2)
       SR=SIN(2*PI*R)/R
       SR1=SIN(2*PI*R1)/R1
       SR2=SIN(2*PI*R2)/R2
       FACR=2*SR*CØS(PI*LP/LMDA)
       RESIST=(((SR1*CA1+SR2*CA2-FACR*CA)*SY)/TERM+(FACR-SR1-SR2)*SZ)*
      1SIN(2*PI*(0.5*LS/LMDA-ABS(S)))/S
       RETURN
       END
```

```
C--------ANTN 1
C-----REQUIRED FØR DIPØLE AND YAGI UDA
      FUNCTIØN REACT(S)
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
      IF (ANTN.LT.9) LS=LP=L
      IF (ANTN.GE.9) LP=2*LHP;LS=2*LHS
      SZ=S*CØS(2*DLPRI)
      SY=-S*SIN(2*DLPRI)
      TERM=YO+SY
      RØW2=(YO+SY)**2
      CA=ZO+SZ
      CA1=CA+0.5*LP/LMDA
      CA2=CA-0.5*LP/LMDA
      R=SQRT(RØW2+CA**2)
      R1=SQRT(RØW2+CA1**2)
      R2=SQRT(RØW2+CA2**2)
      CR=CØS(2*PI*R)/R
      CR1=CØS(2*PI*R1)/R1
      CR2=CØS(2*PI*R2)/R2
      FACX=2*CR*CØS(PI*LP/LMDA)
      REACT=(((CR1*CA1+CA2*CR2-FACX*CA)*SY)/RØW2+(FACX-CR1-CR2)*SZ)*
     1SIN(2*PI*(0.5*LS/LMDA-ABS(S)))/S
      RETURN
      END
```

```
C---------ANTN 2
C-----VERTICAL MØNØPØLE
      SUBRØUTINE BRANCH2
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
 1200 CALL KØSINUS((4*K*L),CC)
      CIN2=ALØG(4*K*L)+.577-CC
      CALL KØSINUS((2*K*L),CC)
      CIN1=ALØG(2*K*L)+.577-CC
      CALL SINUS((4*K*L),SC)
      SIN2=1.57078633+SC
      CALL SINUS((2*K*L),SC)
      SIN1=1.57078633+SC
      RIN=15.*((2.+2*CØS(2*K*L))*CIN1-CØS(2*K*L)*CIN2-2*SIN(2*K*L)*SIN1+
     1SIN(2*K*L)*SIN2)
      RETURN
      END
```

```
C---------ANTN 2
C-----RQRD FOR V MONO, V MONO WITH SCN, INVERTED L, SLOPING LNG WIRE
      SUBROUTINE BRANCH2A
      COMMON   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1COSDL,COSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISOL,K,KAY,KOS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THFPR,VOLTS,VOLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KOS.L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      COMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSION CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VOLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VOLTS,CRNT)
     1,(ZZPAC,ZZPAK)
  200 S3=COS(SIGHV)
      S4=SIN(SIGHV)
      A=COS(K*L*SINDL)-COS(K*L)
      B=SIN(K*L*SINDL)-SINDL*SIN(K*L)
      G=(30.0/COSDL**2)*((A*(1.+CV*S3)+B*CV*S4)**2+
     1(B*(1.-CV*S3)+A*CV*S4)**2)
      IF(N.EQ.1) GV(I)=G
      IF(N.EQ.2) GH(J)=G
      RETURN
      END
```

```
C---------ANTN 2
C-----RQRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      SUBRØUTINE SINUS(X,SC)
      IF(X.GE.10.0)GØ TØ 10
      DX=X/100
      GRAL=0.5
      XA=0.0
      DØ 100 I=2,100
      XA=XA+DX
  100 GRAL=GRAL+SINC(XA)
      GRAL=(GRAL+SINC(X)/2.)*DX
      SC=-3.14159265/2.+GRAL
      GØ TØ 20
   10 SC=-CØS(X)/X
   20 CØNTINUE
      RETURN
      END
```

```
C---------ANTN 2
C-----RGRD FOR V MONO, V MONO WITH SCN, INVERTED L, SLOPING LNG WIRE
      SUBROUTINE KOSINUS(X,CC)
      IF(X.GE.10.0)GO TO 10
      DX=X/100
      GRAL=0.0
      XA=0.0
      DO 100 I=2,100
      XA=XA+DX
  100 GRAL=GRAL+(1.0-COS(XA))/XA
      GRAL=(GRAL+(1.0-COS(X))/2*X)*DX
      CC=ALOG(1.781072*X)-GRAL
      GO TO 20
   10 CC=SIN(X)/X
   20 CONTINUE
      RETURN
      END
```

```
C--------ANTN 2
C-----RQRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      FUNCTIØN SINC(X)
      SINC=SIN(X)/X
      RETURN
      END
```

```
C--------ANTN 3
C-----VERTICAL MONOPOLE WITH GROUND SCREEN
      SUBROUTINE BRANCH3
      COMMON   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1COSDL,COSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISOL,K,KAY,KOS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VOLTS,VOLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KOS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      COMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      COMPLEX ARGP,ARGP2,ARGM,ARGM2,DLTZ1,DLTZ2
      DIMENSION CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VOLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VOLTS,CRNT)
     1,(ZZPAC,ZZPAK)
 1300 C1=SIN(K*L)**2
      IF(C1.LT.WOSQ)C1=WOSQ
      RO=(H**2+L**2)**0.5
      R1=H+RO
      ARGP=CMPLX(0.0,K*L)
      ARGM=CMPLX(0.0,-K*L)
      ARGP2=CMPLX(0.0,2*K*L)
      ARGM2=CMPLX(0.0,-2*K*L)
      DLTZ1=(ADA/4*3.14159265*C1)*(CEXP(ARGP2)*AKEX(-2*K*(RO+L))+
     1CEXP(ARGM2)*AKEX(-2*K*(RO-L))+2*COS(K*L)**2*AKEX(-2*K*H)+
     14*COS(K*L)*AKEX(-K*R1)-4*COS(K*L)*CEXP(ARGM)*AKEX(-K*(R1-L))-
     14*COS(K*L)*CEXP(ARGP)*AKEX(-K*(R1+L)))
      WRITE(6,1311)DLTZ1
 1311 FORMAT('DLTZ1=',F12.6)
      NN=120
      WIRE=.01
      DX=(H-.01)/100
      DUM=.01
      DLTZ2=ZGRAL(DUM)/2.
```

```
      DO 1310 II=2,100
      DUM=DUM+DX
 1310 DLTZ2=DLTZ2+ZGRAL(DUM)
      DLTZ2=(DLTZ2+ZGRAL(H)/2.)*DX
      DLTZ2=-DLTZ2
      WRITE(6,1312)DLTZ2
 1312 FORMAT('DLTZ2=',F12.6)
      CALL KOSINUS((4*K*L),CC)
      CIN2=ALOG(4*K*L)+.577-CC
      CALL KOSINUS((2*K*L),CC)
      CIN1=ALOG(2*K*L)+.577-CC
      CALL SINUS((4*K*L),SC)
      SIN2=1.57078633+SC
      CALL SINUS((2*K*L),SC)
      SIN1=1.57078633+SC
      RIN=15.*((2.+2*COS(2*K*L))*CIN1-COS(2*K*L)*CIN2-2*SIN(2*K*L)*SIN1
     1+SIN(2*K*L)*SIN2)
      RIN=RIN/C1
      RIN=RIN+REAL(DLTZ1+DLTZ2)
      RETURN
      END
```

```
C--------ANTN 3
      SUBRØUTINE BRANCH3A
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WØSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      CØMPLEX GRAL
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
  300 IF((N.EQ.2).AND.(J.GT.1))GØ TØ 310
      S3=CØS(SIGHV)
      S4=SIN(SIGHV)
      A=CØS(K*L*SINDL)-CØS(K*L)
      B=SIN(K*L*SINDL)-SINDL*SIN(K*L)
      C1=SIN(K*L)
      IF((C1.LT.WØSQ).AND.(C1.GE.0.0))C1=WØSQ
      IF((C1.GT.-WØSQ).AND.(C1.LT.0.0))C1=-WØSQ
      C3=A
      IF((C3.LT.WØSQ).AND.(C3.GE.0.0))C3=WØSQ
      IF((C3.GT.-WØSQ).AND.(C3.LT.0.0))C3=-WØSQ
      XB=K*H
      DX=XB/100
      XX=0
      GRAL=PTGRL(XX)/2
      DØ 315 IA=2,100
      XX=XX+DX
  315 GRAL=GRAL+PTGRL(XX)
      GRAL=(GRAL+PTGRL(XB)/2)*DX
      GRAL=1.0-(ADA*SIN(THETA)*GRAL)/120.*3.14159265*C1*C3
```

```
      SRFAC=(CABS(GRAL))**2
  310 CØNTINUE
      G=(30.0/CØSDL**2)*((A*(1.+CV*S3)+B*CV*S4)**2+
     1(B*(1.-CV*S3)+A*CV*S4)**2)*SRFAC/C1**2
      IF(N.EQ.1) GV(I)=G
      IF(N.EQ.2) GH(J)=G
      RETURN
      END
```

```
C---------ANTN 3
C------REQUIRED FOR VERT MONOPOLE WITH GND SCREEN
      FUNCTION AKEX(X)
      COMPLEX AKEX
      XX=ABS(X)
      CALL KOSINUS(XX,CC)
      CALL SINUS(XX,SC)
      IF(X.LT.0.0)AKEX=CMPLX(CC,-SC)
      IF(X.GE.0.0)AKEX=CMPLX(CC,SC)
      RETURN
      END
```

```
C---------ANTN 3
C-----RQRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      SUBRØUTINE SINUS(X,SC)
      IF(X.GE.10.0)GØ TØ 10
      DX=X/100
      GRAL=0.5
      XA=0.0
      DØ 100 I=2,100
      XA=XA+DX
  100 GRAL=GRAL+SINC(XA)
      GRAL=(GRAL+SINC(X)/2.)*DX
      SC=-3.14159265/2.+GRAL
      GØ TØ 20
   10 SC=-CØS(X)/X
   20 CØNTINUE
      RETURN
      END
```

```
C---------ANTN 3
C-----RQRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      SUBRØUTINE KØSINUS(X,CC)
      IF(X.GE.10.0)GØ TØ 10
      DX=X/100
      GRAL=0.0
      XA=0.0
      DØ 100 I=2,100
      XA=XA+DX
  100 GRAL=GRAL+(1.0-CØS(XA))/XA
      GRAL=(GRAL+(1.0-CØS(X))/2*X)*DX
      CC=ALØG(1.781072*X)-GRAL
      GØ TØ 20
   10 CC=SIN(X)/X
   20 CØNTINUE
      RETURN
      END
```

```
C---------ANTN 3
C-----RQRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      FUNCTIØN SINC(X)
      SINC=SIN(X)/X
      RETURN
      END
```

```
C---------ANTN 3
C-----REQUIRED FØR VERT MØNØPØLE WITH GND SCREEN
      FUNCTIØN PTGRL(XX)
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      CØMPLEX PTGRL
      CØMPLEX ARG1,ARG2
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
      S=XX*CØSDL
      IF(S.LE.1)GØ TØ 20
      P1=1+15/(2*(8*S)**2)-(225.*7*9)/(24*(8*S)**4)+(225.*49*81*143)/
     1(720*(8*S)**6)
      Q1=3/(8*S)-315/(6*(8*S)**3)+(9*35*35*99)/(120*(8*S)**6)
      AJ1=(2./(PI*S))**0.5*(P1*CØS(S-3*PI/4)-Q1*SIN(S-3*PI/4))
      GØ TØ 30
   20 CØNTINUE
      AJ1=S/2-S**3/16+S**5/384-S**7/(128*144)+S**9/(512*24*120)
   30 CØNTINUE
      ZZ=(XX**2+K**2*L**2)**0.5
      ARG1=CMPLX(0.0,-ZZ)
      ARG2=CMPLX(0.0,-XX)
      PTGRL=(CEXP(ARG1)-CEXP(ARG2)*CØS(K*L))*AJ1
      RETURN
      END
```

```
C--------ANTN 3
C-----REQUIRED FØR VERT MØNØPØLE WITH GND SCREEN
      FUNCTIØN ZGRAL(XA)
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WØSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      CØMPLEX ADAE,ARG1,ARG2,ZGRAL
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
      ARG1=CMPLX(0.0,-K*(XA**2+L**2)**0.5)
      ARG2=CMPLX(0.0,-K*XA)
      C1=SIN(K*L)**2
      IF(C1.LT.0.01)C1=.01
      XX=(240.*3.14159265**2*XA/(NN*LMDA))*ALØG(XA/(NN*WIRE))
      ADAE=CMPLX(0.0,XX)
      ZGRAL=(ADA*ADAE/(ADA+ADAE))*((CEXP(ARG1)-CEXP(ARG2)*CØS(K*L))/
     1(2*3.14159265*C1*XA))
      RETURN
      END
```

```
C---------ANTN 4
C-----INVERTED L
      SUBRØUTINE BRANCH4
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WØSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
 1400 CALL KØSINUS((2*K*H),CC)
      CI1=CC
      CALL KØSINUS((4*K*H),CC)
      CI2=CC
      CALL SINUS((2*K*H),SC)
      SI1=-SC
      CALL SINUS((4*K*H),SC)
      SI2=-SC
      RIN=60.*(1.41+ALØG(2*L/LMDA)+SINC(2*K*L))+30.*(-0.5*CØS(2*K*H)*
     1(ALØG(2*K*H)+1.270+CI2)+(1.0+CØS(2*K*H))*(ALØG(2*K*H)+0.577-CI1)-
     1SIN(2*K*H)*(0.5*SI2-SI1))
      RETURN
      END
```

```
C--------ANTN 4
      SUBRØUTINE BRANCH4A
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
  400 DENM1=1.0-CØSDL**2*SIN(PHI)**2
      S1=CØS(SIGHH-2*K*H*SINDL)
      S2=SIN(SIGHH-2*K*H*SINDL)
      S3=CØS(SIGHV-2*K*H*SINDL)
      S4=SIN(SIGHV-2*K*H*SINDL)
      A=CØS(K*L)*CØS(K*H*SINDL)-SINDL*SIN(K*L)*SIN(K*H*SINDL)
     1-CØS(K*(H+L))
      B=SINDL*SIN(K*L)*CØS(K*H*SINDL)+CØS(K*L)*SIN(K*H*SINDL)
     1-SINDL*SIN(K*(H+L))
      GI=SIN(K*L*CØSDL*SIN(PHI))-CØSDL*CØS(PHI)*SIN(K*L)
      GR=CØS(K*L*CØSDL*SIN(PHI))-CØS(K*L)
      ETHET=((SIN(PHI)*SINDL*(GR*(1.0-CV*S3)+GI*CV*S4)/DENM1)
     1-(A*(1.0+CV*CØS(SIGHV))+B*CV*SIN(SIGHV))/CØSDL)**2
     1+((SIN(PHI)*SINDL*(GI*(1.0-CV*S3)-GR*CV*S4)/DENM1)
     1-(B*(1.0-CV*CØS(SIGHV))+A*CV*SIN(SIGHV))/CØSDL)**2
      EPHI=(CØS(PHI)/DENM1)**2*((GR*(1.0+CH*S1)-GI*CH*S2)**2
     1+(GI*(1.0+CH*S1)+GR*CH*S2)**2)
      G=30.0*(ETHET+EPHI)
      IF(N.EQ.1) GV(I)=G
      IF(N.EQ.2) GH(J)=G
      RETURN
      END
```

```
C---------ANTN 4
C-----RGRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      SUBRØUTINE SINUS(X,SC)
      IF(X.GE.10.0)GØ TØ 10
      DX=X/100
      GRAL=0.5
      XA=0.0
      DØ 100 I=2,100
      XA=XA+DX
  100 GRAL=GRAL+SINC(XA)
      GRAL=(GRAL+SINC(X)/2.)*DX
      SC=-3.14159265/2.+GRAL
      GØ TØ 20
   10 SC=-CØS(X)/X
   20 CØNTINUE
      RETURN
      END
```

```
C---------ANTN 4
C-----RQRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      SUBRØUTINE KØSINUS(X,CC)
      IF(X.GE.10.0)GØ TØ 10
      DX=X/100
      GRAL=0.0
      XA=0.0
      DØ 100 I=2,100
      XA=XA+DX
  100 GRAL=GRAL+(1.0-CØS(XA))/XA
      GRAL=(GRAL+(1.0-CØS(X))/2*X)*DX
      CC=ALØG(1.781072*X)-GRAL
      GØ TØ 20
   10 CC=SIN(X)/X
   20 CØNTINUE
      RETURN
      END
```

```
C---------ANTN 4
C-----RQRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      FUNCTIØN SINC(X)
      SINC=SIN(X)/X
      RETURN
      END
```

```
C--------ANTN 5
C-----SLØPING LØNG WIRE
      SUBRØUTINE BRANCH5
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
 1500 CALL SINUS((2*K*L),SC)
      SI1=-SC
      CALL KØSINUS((2*K*L),CC)
      CI1=CC
      CALL KØSINUS((4*K*L),CC)
      CI2=CC
      CALL SINUS((4*K*L),SC)
      SI2=-SC
      RIN=30.0*(0.5*(ALØG(K*L)+0.577-CI2)+.693+CØS(K*L)*(CØS(K*L)*
     1(ALØG(K*L)+.577-2*CI1+CI2)-SIN(K*L)*(SI2-2.*SI1)))
      RETURN
      END
```

```
C--------ANTN 5
      SUBRØUTINE BRANCH5A
      CØMMØN    /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
  500 PHI=PHI-DPHIP
      CØSPI=CØS(PHI)
      SINPI=SIN(PHI)
      FCT1=1.0-(SINDL*SINDP+CØSDL*CØSDP*CØSPI)**2
      FCT2=1.0-(CØSDL*CØSDP*CØSPI-SINDL*SINDP)**2
      CIG=(CØS(K*L*(SINDL*SINDP+CØSDL*CØSDP*CØSPI))-CØS(K*L))/FCT1
      SIG=(SIN(K*L*(SINDL*SINDP+CØSDL*CØSDP*CØSPI))-
     1(SINDL*SINDP+CØSDL*CØSDP*CØSPI)*SIN(K*L))/FCT1
      CIGP=(CØS(K*L*(CØSDL*CØSDP*CØSPI-SINDL*SINDP))-CØS(K*L))/FCT2
      SIGP=(SIN(K*L*(CØSDL*CØSDP*CØSPI-SINDL*SINDP))+
     1(SINDL*SINDP-CØSDL*CØSDP*CØSPI)*SIN(K*L))/FCT2
      EPHI1=-CØSDP*SINPI*(CIG+CH*(CIGP*CØS(SIGHH)-SIGP*SIN(SIGHH)))
      EPHI2=-CØSDP*SINPI*(SIG+CH*(CIGP*SIN(SIGHH)+SIGP*CØS(SIGHH)))
      ETHT1=CIG*(CØSDP*CØSPI*SINDL-SINDP*CØSDL)-CV*(CØSDP*CØSPI*SINDL+
     1SINDP*CØSDL)*(CIGP*CØS(SIGHV)-SIGP*SIN(SIGHV))
      ETHT2=SIG*(CØSDP*CØSPI*SINDL-SINDP*CØSDL)-CV*(CØSDP*CØSPI*SINDL+
     1SINDP*CØSDL)*(CIGP*SIN(SIGHV)+SIGP*CØS(SIGHV))
      IF(FCT1.LT.WOSQ)ETHT1=ETHT2=EPHI1=EPHI2=0.0
      G=30.0*(EPHI1**2+EPHI2**2+ETHT1**2+ETHT2**2)
      IF((FCT1.LT.WOSQ).AND.(FCT2.LT.WOSQ))G=.1
      IF(N.EQ.1)GV(I)=G
```

```
IF(N.EQ.2)GH(J)=G
RETURN
END
```

```
C--------ANTN 5
C-----RØRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      SUBRØUTINE SINUS(X,SC)
      IF(X.GE.10.0)GØ TØ 10
      DX=X/100
      GRAL=0.5
      XA=0.0
      DØ 100 I=2,100
      XA=XA+DX
  100 GRAL=GRAL+SINC(XA)
      GRAL=(GRAL+SINC(X)/2.)*DX
      SC=-3.14159265/2.+GRAL
      GØ TØ 20
   10 SC=-CØS(X)/X
   20 CØNTINUE
      RETURN
      END
```

```
C---------ANTN 5
C-----RØRD FØR V MØNØ, V MØNØ WITH SCN, INVERTED L, SLØPING LNG WIRE
      SUBRØUTINE KØSINUS(X,CC)
      IF(X.GE.10.0)GØ TØ 10
      DX=X/100
      GRAL=0.0
      XA=0.0
      DØ 100 I=2,100
      XA=XA+DX
  100 GRAL=GRAL+(1.0-CØS(XA))/XA
      GRAL=(GRAL+(1.0-CØS(X))/2*X)*DX
      CC=ALØG(1.781072*X)-GRAL
      GØ TØ 20
   10 CC=SIN(X)/X
   20 CØNTINUE
      RETURN
      END
```

```
C--------ANTN 6
C-----SLØPING VEE
      SUBRØUTINE BRANCH6
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
 1600 RIN=1.0
      RETURN
      END
```

```
C--------ANTN 6
      SUBROUTINE BRANCH6A
      COMMON   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1COSDL,COSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISOL,K,KAY,KOS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VOLTS,VOLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KOS,L,LH,LHP,LHS,LMDA
      REAL KOS1,KOS2,KOS3,KOS4,KOS5,KOS6,KOS7,KOS8
      INTEGER ANTN,PAR
      COMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSION CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VOLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VOLTS,CRNT)
     1,(ZZPAC,ZZPAK)
  600 ADJ=COS(ALPH)*COSDP
      OPP=SIN(ALPH)
      ALPH=ATAN2(OPP,ADJ)
      COSP=COS(PHI+ALPH)
      COSM=COS(PHI-ALPH)
      KOS1=SINDL*SINDP+COSDL*COSDP*COSM
      KOS2=SINDL*SINDP+COSDL*COSDP*COSP
      KOS3=-SINDL*SINDP+COSDL*COSDP*COSM
      KOS4=-SINDL*SINDP+COSDL*COSDP*COSP
      KOS5=COSDL*SINDP+SINDL*COSDP*COSM
      KOS6=COSDL*SINDP+SINDL*COSDP*COSP
      KOS7=-COSDL*SINDP+SINDL*COSDP*COSM
      KOS8=-COSDL*SINDP+SINDL*COSDP*COSP
      U1=K*L*(1.0-KOS1)
      U2=K*L*(1.0-KOS2)
      U3=K*L*(1.0-KOS3)
      U4=K*L*(1.0-KOS4)
      S1=COS(SIGHH-2*K*H*SINDL)
      S2=SIN(SIGHH-2*K*H*SINDL)
      S3=COS(SIGHV-2*K*H*SINDL)
```

```
      S4=SIN(SIGHV-2*K*H*SINDL)
      CØSU1=CØS(U1)
      CØSU2=CØS(U2)
      CØSU3=CØS(U3)
      CØSU4=CØS(U4)
      SINU1=SIN(U1)
      SINU2=SIN(U2)
      SINU3=SIN(U3)
      SINU4=SIN(U4)
      A=(KØS7*(CØSU1-1.)/U1-KØS8*(CØSU2-1.)/U2)+CABS(RV)*((KØS6*(
     1(CØSU4-1.)*S3+SINU4*S4)/U4)-KØS5*((CØSU3-1.)*S3+SINU3*S4)/U3)
      B=(KØS8*SINU2/U2-KØS7*SINU1/U1)+CABS(RV)*(KØS5*(SINU3*S3-(CØSU3-1.
     1)*S4)/U3+KØS6*((CØSU4-1.)*S4-SINU4*S3)/U4)
      C=SIN(PHI+ALPH)*(CØSU2-1.)/U2-SIN(PHI-ALPH)*(CØSU1-1.)/U1
     1+CABS(RH)*((SIN(PHI+ALPH)*(CØSU4-1.)/U4-SIN(PHI-ALPH)*(CØSU3-1.)
     1/U3)*S1-(SIN(PHI-ALPH)*SINU3/U3-SIN(PHI+ALPH)*SINU4/U4)*S2)
      Y=SIN(PHI-ALPH)*SINU1/U1-SIN(PHI+ALPH)*SINU2/U2+CABS(RH)*((SIN(PHI
     1-ALPH)*SINU3/U3-SIN(PHI+ALPH)*SINU4/U4)*S1+(SIN(PHI+ALPH)*
     1(CØSU4-1.)/U4-SIN(PHI-ALPH)*(CØSU3-1.)/U3)*S2)
      G=0.05*(A**2+B**2+CØSDP**2*(C**2+Y**2))
      IF(N.EQ.1) GV(I)=G
      IF(N.EQ.2) GH(J)=G
      RETURN
      END
```

```
C--------ANTN 7
C-----HØRIZØNTAL RHØMBIC
      SUBRØUTINE BRANCH7
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
 1700 RIN=1.0
      RETURN
      END
```

```
C--------ANTN 7
      SUBRØUTINE BRANCH7A
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
  700 SINAC=SIN(ALPCM)
      CØSAC=CØS(ALPCM)
      CØSPI=CØS(PHI)
      SINPI=SIN(PHI)
      S1=CØS(SIGHH-2*K*H*SINDL)
      S3=CØS(SIGHV-2*K*H*SINDL)
      U1=1.0-CØSDL*(SINAC*CØSPI+CØSAC*SINPI)
      U2=1.0-CØSDL*(SINAC*CØSPI-CØSAC*SINPI)
      G=2.16*((CØSAC*SIN(K*0.5*L*U1)*SIN(K*0.5*L*U2)/(U1*U2))**2)*
     1(((CØSPI-SINAC*CØSDL)**2)*((CABS(RH))**2+1.0+2.0*(CABS(RH))*S1)+
     1(SINDL**2)*(SINPI**2)*((CABS(RV))**2+1.0-2.0*(CABS(RV))*S3))
      IF(N.EQ.1) GV(I)=G
      IF(N.EQ.2) GH(J)=G
      RETURN
      END
```

```
C---------ANTN 8
C-----VERTICAL HALF RHØMBIC
      SUBRØUTINE BRANCH8
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
 1800 RIN=1.0
      RETURN
      END
```

```
C--------ANTN 8
      SUBRØUTINE BRANCH8A
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WØSQ,WYE,XGRAL,Y0,Z,Z0,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
  800 CØSAC=CØS(ALPCM)
      SINAC=SIN(ALPCM)
      FACK1=1.0-CØSDL*CØSAC*CØSPI-SINDL*SINAC
      FACK2=1.0-CØSDL*CØSAC*CØSPI+SINDL*SINAC
      UU1=CØS(SIGHH-2*K*H*SINDL)
      UU2=SIN(SIGHH-2*K*H*SINDL)
      UU3=CØS(SIGHV-2*K*H*SINDL)
      UU4=SIN(SIGHV-2*K*H*SINDL)
      S1=SIN(K*L*FACK1)
      CE1=CØS(K*L*FACK1)
      S2=SIN(K*L*FACK2)
      CE2=CØS(K*L*FACK2)
      R1=(1.0-CE1)/FACK1
      AI1=S1/FACK1
      R2=(CE1*(1.0-CE2)+S1*S2)/FACK2
      AI2=(CE1*S2-S1*(1.0-CE2))/FACK2
      R3=(1.0-CE1)*CØS(2*K*L*SINAC*SINDL)-(1.0-CE1)*SIN(2*K*L*SINAC
     1*SINDL)
      F1=(AI3*CE1-R3*S1)/FACK1
      F2=(R3*CE1+AI3*S1)/FACK1
      F3=(1.0-CE2)/FACK2
```

```
 F4=S2/FACK2
 RB=R1+R2+CABS(RV)*((F2+F3)*UU3-(F1+F4)*UU4)
 BI=AI1+AI2-CABS(RV)*((F2+F3)*UU4+(F1+F4)*UU3)
 RC=R2-R1+CABS(RV)*((F2-F3)*UU3-(F1-F4)*UU4)
 CC=AI2-AI1+CABS(RV)*((F2-F3)*UU4+(F1-F4)*UU3)
 RA=R1+R2+CABS(RH)*((F2+F3)*UU1+(F1+F4)*UU2)
 A1=AI1+AI2+CABS(RH)*((F2+F3)*UU2+(F1+F4)*UU1)
 G=0.1*((RB*CØSAC*CØSPI*SINDL+RC*SINAC*CØSDL)**2
1+(BI*CØSAC*CØSPI*SINDL+CC*SINAC*CØSDL)**2+(RA*CØSAC*SINPI)**2
1+(A1*CØSAC*SINPI)**2)
 IF(N.EQ.1) GV(I)=G
 IF(N.EQ.2) GH(J)=G
 RETURN
 END
```

```
C---------ANTN 9
C-----YAGI UDA
      SUBROUTINE BRANCH9
      COMMON   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1COSDL,COSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISOL,K,KAY,KOS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VOLTS,VOLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KOS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      COMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      COMPLEX CUR,ABVER,ABHOR,ZIN,EJ,EK
      DIMENSION CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VOLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VOLTS,CRNT)
     1,(ZZPAC,ZZPAK)
1900  DO 1920 I=1,5
      DO 1910 J=1,5
      Z(I,J)=CMPLX (0.,0.)
1910  CONTINUE
      CUR(I)=CMPLX(0.,C.)
1920  CONTINUE
      PI=3.14159265
C-----IMP FOR SELF OF ISOLATED YAGI
      ISOL=1
      DO 1930 I=1,NE
      IA=I
      IF(LH(1).EQ.0.0) IA=I+1
      CUMDIS=(SQRT(2.)/200.*LH(IA))*T
1925  LHS=LHP=LH(IA)
      CALL ZINT
      Z(I,I)=CMPLX(RGRAL,XGRAL)
1930  CONTINUE
C-----IMP FOR MUTUAL OF ISOLATED YAGI
      ISOL=1
```

```
      DØ 1931 I=1,NE-1
      IE=NE-I
      CUMDIS=0.
      MA=I
      IA=I
      MAA=MA
      IF(LH(1).EQ.0.0) IA=I+1;MAA=MA+1
      DØ 1931 J=1,IE
      MA=MA+1
      MAA=MAA+1
      CUMDIS=CUMDIS+D(MAA)
      LHP=LH(IA)
      LHS=LH(MAA)
      CALL ZINT
      Z(I,MA)=CMPLX(RGRAL,XGRAL)
      Z(MA,I)=Z(I,MA)
1931  CØNTINUE
C-----IMP FØR MUTUAL ØF ISØL/IMAGE ØF YAGI
      ISØL=0
      WYE(1)=0.
      DØ 1932 I=1,NE-1
      IE=NE-I
      DLEG=0.
      MA=I
      IA=I
      MAA=MA
      IF(LH(1).EQ.0.0) IA=I+1;MAA=MA+1
      DØ 1932 J=1,IE
      MA=MA+1
      MAA=MAA+1
      DLEG=DLEG+D(MAA)
      DLEG2=DLEG**2
      IF(I.EQ.1)WYE(MA)=DLEG
      HDBL=2.*(H+WYE(I)*SIN(ALPH))
      HDBL2=HDBL**2
```

```
      CUMDIS=SQRT(HDBL2+DLEG2-2.*HDBL*DLEG*CØS(PI/2.+ALPH))
      LHP=LH(IA)
      LHS=LH(MAA)
      CALL ZINT
      Z(I,MA)=Z(I,MA)+CEE*CMPLX(RGRAL,XGRAL)
      Z(MA,I)=Z(I,MA)
1932  CØNTINUE
C-----IMP FØR MUTUAL ØF ISØL/IMAGE ØF YAGI (2ND PART)
      DØ 1933 I=1,NE
      CUMDIS=2.*(H+WYE(I)*SIN(ALPH))
      IA=I
      IF(LH(1).EQ.0.0) IA=I+1
      LHS=LHP=LH(IA)
      CALL ZINT
      Z(I,I)=Z(I,I)+CEE*CMPLX(RGRAL,XGRAL)
1933  CØNTINUE
      WRITE (6,1934) Z(2,2)
1934  FØRMAT($SELF ØF DRIVER: Z=$,2F12.1)
C-----PACK MATRIX INCIDENT TØ SØLUTIØN ØF MATRIX FØR CURRENT VECTØR
      DØ 1940 I=1,NE
      DØ 1940 J=1,NE
1940  ZZPAC(I,J)=REAL(Z (I,J))
      DØ 1941 I=NE+1,2*NE
      DØ 1941 J=NE+1,2*NE
1941  ZZPAC(I,J)=REAL(Z (I-NE,J-NE))
      DØ 1942 I=1,NE
      DØ 1942 J=NE+1,2*NE
      ZZPAC(I,J)=-AIMAG(Z (I,J-NE))
1942  ZZPAC(J,I)=AIMAG(Z (J-NE,I))
C-----PLACE MATRIX INTØ A CØLUMN-STACKED VECTØR MATRIX
      LQ=0
      DØ 1943 IQ=1,2*NE
      DØ 1943 JQ=1,2*NE
      LQ=LQ+1
      ZZPAK(LQ)=ZZPAC(JQ,IQ)
```

```
1943  CØNTINUE
      VØLDRI=VØLTS(2)
      IF (LH(1).EQ.0.0) VØLTS(1)=VØLDRI;VØLTS(2)=0.0
      CALL SIMQ (ZZPAK,VØLTS,2*NE,KS)
      IF (KS.EQ.1) WRITE (6,1950)
1950  FØRMAT(1HO,$FRØM SUBRØUTINE SIMQ KS=1-CURRENT SØL HAS NØ MEANING$)
C-----CURRENT MAXIMUM (CØMPLEX)
      DØ 1960 I=1,NE
      IA=I
      IF (LH(1).EQ.0.0) IA=I+1
      ELEM=LH(IA)/LMDA
      IF((ELEM.GT..991).AND.(ELEM.LT.1.009))ELEM=.99
      IF((ELEM.GT..491).AND.(ELEM.LT..509))ELEM=.49
      CUR(I)=(CMPLX(CRNT(I),CRNT(I+NE)))*1000.
      IF((LH(1).EQ.0.).AND.(I.EQ.1)) CURDRI=CUR(1)/1000.
      IF((LH(1).NE.0.).AND.(I.EQ.2)) CURDRI=CUR(2)/1000.
      CUR(I)=CUR(I)/SIN(2.*PI*ELEM)
      BET=(180./PI)*ATAN2(AIMAG(CUR(I)), REAL(CUR(I)))
      CURMAG=CABS(CUR(I))
1960  CURMAG=CABS(CUR(I))
      ZIN=VØLDRI/CURDRI
      RIN=REAL(ZIN)
      WRITE (6,1963) AIMAG(ZIN)
1963  FØRMAT ($XIN=$,F12.1)
      RETURN
      END
```

```
C--------ANTN 9
      SUBROUTINE BRANCH9A
      COMMON   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1COSDL,COSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISOL,K,KAY,KOS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VOLTS,VOLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KOS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      COMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      COMPLEX ABVER,ABHOR,EJ,EK,REV,REH,ZIN
      DIMENSION CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VOLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VOLTS,CRNT)
     1,(ZZPAC,ZZPAK)
900   KOS1=COS(THETA)*SIN(ALPH)+SIN(THETA)*SIN(PHI)*COS(ALPH)
      ABVER=ABHOR=CMPLX(0.,0.)
      DO 910 IA=1,NE
      WK=K*WYE(IA)*KOS1
      EJ=CMPLX(COS(WK),SIN(WK))
      S1=COS(-2*K*(H+WYE(IA)*SIN(ALPH))*COS(THETA))
      S2=SIN(-2*K*(H+WYE(IA)*SIN(ALPH))*COS(THETA))
      EK=CMPLX(S1,S2)
      IF(THTEM.LT..01)REV=1.+RV*EK;REH=0.;GO TO 905
      REV=1.-RV*EK
      REH=1.+RH*EK
905   FCT=COS(K*LH(IA)*KOS)-COS(K*LH(IA))
      ABVER=ABVER+(CUR(IA)/1000.)*EJ*FCT*REV
      ABHOR=ABHOR+(CUR(IA)/1000.)*EJ*FCT*REH
910   CONTINUE
      CTP=COS(THETA)*COS(PHI)
      SP=SIN(PHI)
      IF(THTEM.LT..01)ETHET=60.*CABS(ABVER/SIN(THETA));EPHI=0.;GO TO 912
      ETHET=60.*CABS(CTP/SINSQ*ABVER)
      EPHI =60.*CABS(SP /SINSQ*ABHOR)
```

```
912   G=(ETHET**2+EPHI**2)/(30*CABS(CURDRI)**2)
      IF(N.EQ.1) GV(I)=G
      IF(N.EQ.2) GH(J)=G
C-----REINITIALIZE THE VØLTS ARRAY FØR NEXT CHØICE ARRAY
      IF((N.EQ.2).AND.(J.EQ.360)) GØ TØ 915
      RETURN
915   VØLTS(1)=0.0
      VØLTS(2)=VØLDRI
      DØ 920 IA=3,10
920   VØLTS(IA)=0.0
      RETURN
      END
```

```
C---------ANTN 9
C-----REQUIRED FØR DIPØLE AND YAGI UDA
       SUBRØUTINE ZINT
       CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
      1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
      2,L,LH,LMDA,LHP,LHS,M,N,NE,VN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
      3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
      4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
       REAL K,KØS,L,LH,LHP,LHS,LMDA
       INTEGER ANTN,PAR
       CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
       DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
      1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
       EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
      1,(ZZPAC,ZZPAK)
       IF (ISØL.EQ.0) GØ TØ 1940
       SVDLP=DLPRI
       DLPRI=0.
1940   YO=CUMDIS*CØS(DLPRI)/LMDA
       ZO=-CUMDIS*SIN(DLPRI)/LMDA
       S=-LHS/LMDA
       RGRAL=.5*RESIST(S)
       DS=LHS/(50.*LMDA)
       DØ 1950 N=2,100
       S=S+DS
1950   RGRAL=RGRAL+RESIST(S)
       RGRAL=-30.*(RGRAL+.5*RESIST(LHS/LMDA))*DS
       S=-LHS/LMDA
       XGRAL=.5*REACT(S)
       DØ 1960 N=2,100
       S=S+DS
1960   XGRAL=XGRAL+REACT(S)
       XGRAL=-30.*(XGRAL+.5*REACT(LHS/LMDA))*DS
       IF (ISØL.EQ.1) DLPRI=SVDLP
       RETURN
       END
```

```
C-----REQUIRED FOR DIPOLE AND YAGI UDA
      FUNCTION RESIST(S)
      COMMON   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1COSDL,COSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISOL,K,KAY,KOS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VOLTS,VOLDRI,WIRE,WOSQ,WYE,XGRAL,YO,Z,ZO,ZZPAK
      REAL K,KOS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      COMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSION CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VOLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VOLTS,CRNT)
     1,(ZZPAC,ZZPAK)
      IF (ANTN.LT.9) LS=LP=L
      IF (ANTN.GE.9) LP=2*LHP;LS=2*LHS
      SZ=S*COS(2*DLPRI)
      SY=-S*SIN(2*DLPRI)
      TERM=YO+SY
      ROW2=(YO+SY)**2
      CA=ZO+SZ
      CA1=CA+0.5*LP/LMDA
      CA2=CA-0.5*LP/LMDA
      R=SQRT(ROW2+CA**2)
      R1=SQRT(ROW2+CA1**2)
      R2=SQRT(ROW2+CA2**2)
      SR=SIN(2*PI*R)/R
      SR1=SIN(2*PI*R1)/R1
      SR2=SIN(2*PI*R2)/R2
      FACR=2*SR*COS(PI*LP/LMDA)
      RESIST=(((SR1*CA1+SR2*CA2-FACR*CA)*SY)/TERM+(FACR-SR1-SR2)*SZ)*
     1SIN(2*PI*(0.5*LS/LMDA-ABS(S)))/S
      RETURN
      END
```

```
C---------ANTN 9
C-----REQUIRED FØR DIPØLE AND YAGI UDA
      FUNCTIØN REACT(S)
      CØMMØN   /IMP/ A,ADA,ALPH,ALTEM,ANTN,B,C,CEE,CH,CV,CURDRI,CUMDIS,
     1CØSDL,CØSDP,D,DELTA,DLPRI,DPHIP,G,GV,GH,H,HTEMP,I,J,ISØL,K,KAY,KØS
     2,L,LH,LMDA,LHP,LHS,M,N,NE,NN,PAR,PHI,PHIPR,PI,RIN,RV,RH,RGRAL,
     3RVPRI,RHPRI,SIGHH,SIGHV,SINSQ,SINDL,SINDP,S1,S2,S3,S4,T,THETA,
     4THEPR,VØLTS,VØLDRI,WIRE,WOSQ,WYE,XGRAL,Y0,Z,Z0,ZZPAK
      REAL K,KØS,L,LH,LHP,LHS,LMDA
      INTEGER ANTN,PAR
      CØMPLEX ADA,CEE,CURDRI,RV,RH,RVPRI,RHPRI,Z
      DIMENSIØN CRNT(10),CUR(10),D(4),FAC1(180),FAC2(180),GV(90),GH(360)
     1,LH(5),VØLTS(10),WYE(5),Z(5,5),ZZPAC(10,10),ZZPAK(100)
      EQUIVALENCE (GH(1),FAC1(1)),(GH(181),FAC2(1)),(VØLTS,CRNT)
     1,(ZZPAC,ZZPAK)
      IF (ANTN.LT.9) LS=LP=L
      IF (ANTN.GE.9) LP=2*LHP;LS=2*LHS
      SZ=S*CØS(2*DLPRI)
      SY=-S*SIN(2*DLPRI)
      TERM=Y0+SY
      RØW2=(Y0+SY)**2
      CA=Z0+SZ
      CA1=CA+0.5*LP/LMDA
      CA2=CA-0.5*LP/LMDA
      R=SQRT(RØW2+CA**2)
      R1=SQRT(RØW2+CA1**2)
      R2=SQRT(RØW2+CA2**2)
      CR=CØS(2*PI*R)/R
      CR1=CØS(2*PI*R1)/R1
      CR2=CØS(2*PI*R2)/R2
      FACX=2*CR*CØS(PI*LP/LMDA)
      REACT=(((CR1*CA1+CA2*CR2-FACX*CA)*SY)/RØW2+(FACX-CR1-CR2)*SZ)*
     1SIN(2*PI*(0.5*LS/LMDA-ABS(S)))/S
      RETURN
      END
```

```
C--------ANTN 9
C-----REQUIRED FØR YAGI
      SUBRØUTINE SIMQ(A,B,N,KS)
      DIMENSIØN A(1),B(1)
C        FØRWARD SØLUTIØN
      TØL=0.0
      KS=0
      JJ=-N
      DØ 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0
      IT=JJ-J
      DØ 30 I=J,N
C        SEARCH FØR MAXIMUM CØEFFICIENT IN CØLUMN
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CØNTINUE
C        TEST FØR PIVØT LESS THAN TØLERANCE (SINGULAR MATRIX)
      IF(ABS(BIGA)-TØL) 35,35,40
   35 KS=1
      RETURN
C        INTERCHANGE RØWS IF NECESSARY
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DØ 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
C        DIVIDE EQUATIØN BY LEADING CØEFFICIENT
   50 A(I1)=A(I1)/BIGA
```

```
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
C          ELIMINATE NEXT VARIABLE
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DØ 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DØ 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
C          BACK SØLUTIØN
   70 NY=N-1
      IT=N*N
      DØ 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DØ 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

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13. ABSTRACT

An arbitrarily orientable Yagi-Uda array antenna was modeled, and a computer simulation run to obtain the input impedance, gain pattern and front-to-back ratio of various arrays. The model made provisions for the antenna to be operated over either a lossy ground plane or aboard a ship in seas of specified state. Quick solution turn-around, with CRT display, enabled relatively rapid optimization of numerous arrays.

Theory, resultant optimal designs and performances, photographs, and program listing are included.

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